thm_2Erat_2ERAT__SGN__ALT (TMbsHL6vz47Rthyn5F5MCm2LSfHsZCJkZN2)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.\mathsf{c\_2Ebool\_2ET})$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 4 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 5 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{7}$$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{9}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{10}$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{11}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{12}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{13}$$

Definition 12 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 13 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{14}$$

Definition 14 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Definition 15 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 16 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 17 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 19 *We define* c_2EintExtension_2ESGN *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Eboo$

Definition 20 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

Definition 21 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x$.

Definition 22 *We define* c_2Emarker_2ECong *to be* $\lambda V0x \in 2.V0x$.

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{15}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{16}$$

Definition 23 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

Definition 24 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 25 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 26 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 27 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 28 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{17}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{18}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 29 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 30 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 31 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{20}$$

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{21}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{22}$$

Definition 32 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ t$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{23}$$

Definition 33 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{24}$$

Definition 34 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Definition 35 *We define* c_2Erat_2Erat__equiv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 36 *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{25}$$

Definition 37 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \quad (26)$$

Definition 38 *We define* c_2Efrac_2Efrac__0 *to be* $(ap\ c_2Efrac_2Eabs__frac\ (ap\ (ap\ ($c_2Epair_2E_2C ty_2

Definition 39 *We define* c_2Efrac_2Efrac__sgn *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap$ c_2EintExtension_2ES

Let $ty\_2Erat\_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \quad (27)$$

Let $c\_2Erat\_2Erep\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Erep\_\_rat\_\_CLASS \in ((2^{ty\_2Efrac\_2Efrac})^{ty\_2Erat\_2Erat}) \quad (28)$$

Definition 40 *We define* c_2Erat_2Erep__rat *to be* $\lambda V0a \in ty_2Erat_2Erat.(ap\ ($c_2Emin_2E_40 $ty\_2Efrac\_$

Definition 41 *We define* c_2Efrac_2Efrac__ainv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_fr$

Let $c\_2Erat\_2Eabs\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Eabs\_\_rat\_\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \quad (29)$$

Definition 42 *We define* c_2Erat_2Eabs__rat *to be* $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_\_rat\_\_C$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \quad (30)$$

Definition 43 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Definition 44 *We define* c_2Efrac_2Efrac__add *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Definition 45 *We define* c_2Efrac_2Efrac__sub *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Definition 46 *We define* c_2Erat_2Erat__sub *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 47 *We define* c_2Erat_2Erat__sgn *to be* $\lambda V0r \in ty\_2Erat\_2Erat.(ap$ c_2Efrac_2Efrac__sgn $(ap$ c_2

Definition 48 *We define* c_2Erat_2Erat__les *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap\ ($

Definition 49 *We define* c_2Erat_2Erat__gre *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 50 *We define* c_2Efrac_2Efrac__mul *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Definition 51 *We define* c_2Erat_2Erat__mul *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 52 *We define* c_2Einteger_2EABS *to be* $\lambda V0n \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ ($c_2Ebool_2EC

Definition 53 *We define* c_2Efrac_2Efrac__minv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_f$

Definition 54 *We define* c_2Erat_2Erat__minv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap$ c_2Erat_2Eabs__rat $(ap$ c_

Definition 55 *We define* c_2Erat_2Erat__leq *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$ (

Definition 56 *We define* c_2Erat_2Erat__add *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 57 *We define* c_2Erat_2Erat__0 *to be* $(ap$ c_2Erat_2Eabs__rat c_2Efrac_2Efrac__0$)$.

Definition 58 *We define* c_2Efrac_2Efrac__1 *to be* $(ap\ c_2Efrac_2Eabs__frac\ (ap\ (ap\ ($c_2Epair_2E_2C ty_2

Definition 59 *We define* c_2Erat_2Erat__1 *to be* $(ap$ c_2Erat_2Eabs__rat c_2Efrac_2Efrac__1$)$.

Definition 60 *We define* c_2Einteger_2ENum *to be* $\lambda V0i \in ty_2Einteger_2Eint.(ap\ ($c_2Emin_2E_40 ty_2E

Definition 61 *We define* c_2Erat_2Erat__ainv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap$ c_2Erat_2Eabs__rat $(ap$ c_2

Definition 62 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Let $c\_2Erat\_2ERATD : \iota$ be given. Assume the following.

$$c\_2Erat\_2ERATD \in (ty\_2Enum\_2Enum^{ty\_2Erat\_2Erat}) \tag{31}$$

Let $c\_2Erat\_2ERATN : \iota$ be given. Assume the following.

$$c\_2Erat\_2ERATN \in (ty\_2Einteger\_2Eint^{ty\_2Erat\_2Erat}) \tag{32}$$

Definition 63 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 64 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 65 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Earithmetic\_2Enum\_\_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2Enum\_\_CASE\ A\_27a \in (((A\_27a^{(A\_27a^{ty\_2Enum\_2Enum})})^{A\_27a})^{ty\_2Enum\_2Enum}) \tag{33}$$

Definition 66 *We define* c_2Erelation_2EWF *to be* $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ ($c_2Ebool_2E_21

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{34}$$

Definition 67 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 68 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 69 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 70 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 71 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 72 *We define* c_2Erat_2Erat__of__num *to be* $(ap\ (ap\ ($c_2Erelation_2EWFREC ty_2Enum_2Enum

Definition 73 *We define* c_2Erat_2Erat__of__int *to be* $\lambda V0i \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Definition 74 *We define* c_2Efrac_2Efrac__div *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 75 *We define* c_2Erat_2Erat__div *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Assume the following.

$$((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \tag{35}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{36}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((\neg(V0n = \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n)))) \tag{37}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n)))\Leftrightarrow(V0n = \mathsf{c\_2Enum\_2E0}))) \tag{38}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{39}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m))))) \tag{40}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))))))))))) \tag{41}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{42}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((V0m = V1n) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m)))))) \tag{43}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{44}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m))))) \tag{45}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(V0m = V1n)) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n)) \vee (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m)))))) \tag{46}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap \; \mathsf{c\_2Enum\_2ESUC} \; V0n) = (ap \; (ap \; c\_2Earithmetic\_2E\_2B \; (ap \; \mathsf{c\_2Earithmetic\_2ENUMERAL} \; (ap \; \mathsf{c\_2Earithmetic\_2EBIT1} \; \mathsf{c\_2Earithmetic\_2EZERO}))) \; V0n))) \tag{47}$$

Assume the following.

$$True \tag{48}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \; V0t1) \Rightarrow (p \; V1t2)) \Rightarrow (((p \; V1t2) \Rightarrow (p \; V0t1)) \Rightarrow ((p \; V0t1) \Leftrightarrow (p \; V1t2)))))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \; V0t))) \tag{50}$$

Assume the following.

$$(\forall V0t \in 2.((p \; V0t) \vee (\neg (p \; V0t)))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty \; A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p \; V0t)) \Leftrightarrow (p \; V0t))) \tag{52}$$

Assume the following.

$$(\forall V0t \in 2.(((p \; V0t) \Rightarrow False) \Rightarrow (\neg (p \; V0t)))) \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p \; V0t)) \Rightarrow ((p \; V0t) \Rightarrow False))) \tag{54}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \; V0t)) \Leftrightarrow (p \; V0t)) \wedge ((((p \; V0t) \wedge True) \Leftrightarrow (p \; V0t)) \wedge (((False \wedge (p \; V0t)) \Leftrightarrow False) \wedge ((((p \; V0t) \wedge False) \Leftrightarrow False) \wedge (((p \; V0t) \wedge (p \; V0t)) \Leftrightarrow (p \; V0t))))))) \tag{55}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \; V0t)) \Leftrightarrow True) \wedge ((((p \; V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \; V0t)) \Leftrightarrow (p \; V0t)) \wedge ((((p \; V0t) \vee False) \Leftrightarrow (p \; V0t)) \wedge (((p \; V0t) \vee (p \; V0t)) \Leftrightarrow (p \; V0t))))))) \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \; V0t)) \Leftrightarrow (p \; V0t)) \wedge ((((p \; V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \; V0t)) \Leftrightarrow True) \wedge ((((p \; V0t) \Rightarrow (p \; V0t)) \Leftrightarrow True) \wedge (((p \; V0t) \Rightarrow False) \Leftrightarrow (\neg (p \; V0t)))))))) \tag{57}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{60}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1)\wedge((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{63}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((\neg((p\ V0A)\Rightarrow(p\ V1B)))\Leftrightarrow((p\ V0A)\wedge (\neg(p\ V1B)))))) \tag{64}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{65}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\vee(p\ V1B))\Leftrightarrow((p\ V1B)\vee (p\ V0A))))) \tag{66}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{67}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{68}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{69}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.(((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{72}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v)) \Leftrightarrow True)) \tag{73}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow ((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V0a)))) \tag{74}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow ((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V1b)))) \tag{75}$$

Assume the following.

$$((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_ainv}\ \mathsf{c\_2Efrac\_2Efrac\_\_0}) = \mathsf{c\_2Efrac\_2Efrac\_\_0}) \quad (76)$$

Assume the following.

$$(\forall V0f1 \in ty\_2Efrac\_2Efrac.((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_ainv}\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_ainv}\ V0f1)) = V0f1)) \quad (77)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = V0x)) \quad (78)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x)) = V0x)) \quad (79)$$

Assume the following.

$$(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \quad (80)$$

Assume the following.

$$((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \quad (81)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))))) \quad (82)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))))) \quad (83)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (84)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\Leftrightarrow((\neg(V0n = \mathsf{c\_2Enum\_2E0}))\vee(\neg(V1m = \mathsf{c\_2Enum\_2E0}))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m))))\Leftrightarrow False)))))) \tag{85}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.((\exists V1n \in ty\_2Enum\_2Enum. ((V0p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\wedge(\neg(V1n = \mathsf{c\_2Enum\_2E0}))))\vee ((\exists V2n \in ty\_2Enum\_2Enum.((V0p = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V2n)))\wedge(\neg(V2n = \mathsf{c\_2Enum\_2E0}))))\vee (V0p = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))) \tag{86}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Einteger\_2ENum}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)) = V0n)) \tag{87}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge (((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow False)\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))))))))))))) \tag{88}$$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\Leftrightarrow(V0m = V1n))))\wedge((\forall V2x \in ty\_2Einteger\_2Eint.(\forall V3y \in ty\_2Einteger\_2Eint.(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V2x) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V3y))\Leftrightarrow(V2x = V3y))))\wedge(\forall V4n \in ty\_2Enum\_2Enum.(\forall V5m \in ty\_2Enum\_2Enum.((((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4n) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V5m)))\Leftrightarrow((V4n = \mathsf{c\_2Enum\_2E0})\wedge(V5m = \mathsf{c\_2Enum\_2E0})))\wedge(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V4n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V5m))\Leftrightarrow((V4n = \mathsf{c\_2Enum\_2E0})\wedge(V5m = \mathsf{c\_2Enum\_2E0}))))))))) \tag{89}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{90}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0}))\wedge(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n))\Rightarrow(p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{91}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ \mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\Leftrightarrow False)\wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO})\Leftrightarrow False)\wedge(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO})\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))\Leftrightarrow(V0n = V1m))\wedge(((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))\Leftrightarrow(V0n = V1m))))))))))) \tag{93}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))))))))) \tag{94}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\Leftrightarrow(V0m = V1n)))) \tag{95}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Enum\_2E0})))) \tag{96}$$

Assume the following.

$$(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ ty\_2Efrac\_2Efrac\ ty\_2Erat\_2Erat)\ \mathsf{c\_2Erat\_2Erat\_\_equiv})\ \mathsf{c\_2Erat\_2Eabs\_\_rat})\ \mathsf{c\_2Erat\_2Erep\_\_rat})) \tag{97}$$

Assume the following.

$$((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ \mathsf{c\_2Efrac\_2Efrac\_\_0})) \tag{98}$$

Assume the following.

$$(\forall V0f1 \in ty\_2Efrac\_2Efrac.((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_sgn}\ (ap\ \mathsf{c\_2Erat\_2Erep\_\_rat}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V0f1))) = (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_sgn}\ V0f1))) \tag{99}$$

Assume the following.

$$(\forall V0x \in ty\_2Efrac\_2Efrac.((ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erep\_\_rat}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V0x)))) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_ainv}\ V0x)))) \tag{100}$$

Assume the following.

$$(\forall V0f1 \in ty\_2Efrac\_2Efrac.(\forall V1f2 \in ty\_2Efrac\_2Efrac.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V0f1))\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V1f2)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ V0f1))\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ V1f2)))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ V1f2))\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ V0f1))))))) \tag{101}$$

Assume the following.

$$(\forall V0f1 \in ty\_2Efrac\_2Efrac.(\forall V1f2 \in ty\_2Efrac\_2Efrac.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V0f1))\ (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ V1f2)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ V0f1))\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ V1f2)))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ V1f2))\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ V0f1))))))) \tag{102}$$

Assume the following.

$$(\forall V0n1 \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V0n1) = (ap\ \mathsf{c\_2Erat\_2Eabs\_\_rat}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n1))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))) \tag{103}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V0a))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V1b)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0a)\ V1b))))) \tag{104}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0a)\ V1b) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V1b)\ V0a)))) \tag{105}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sub}\ V0r1)\ V1r2) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{106}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ V0r1)\ V1r2) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0r1)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_minv}\ V1r2))))) \tag{107}$$

Assume the following.

$$((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) \tag{108}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V1r2)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{109}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.(((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1) = V1r2) \Leftrightarrow (V0r1 = (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))))) \tag{110}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.(((ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2)) \Leftrightarrow (V0r1 = V1r2)))) \tag{111}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.((((ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ V0r1) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V0r1)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))) \land ((((ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ V0r1) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \Leftrightarrow (V0r1 = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))) \land (((ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ V0r1) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_gre}\ V0r1)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))))))) \tag{112}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1))) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ V0r1))) \tag{113}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_mul}\ V0r1)\ V1r2)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ V0r1))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ V1r2))))) \tag{114}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.((\neg(V0r1 = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))\Rightarrow((ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_minv}\ V0r1)) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ V0r1)))) \tag{115}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V0r1)\ V1r2))\Rightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V1r2)\ V0r1)))))) \tag{116}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ V0r1)\ V0r1))) \tag{117}$$

Assume the following.

$$(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \tag{118}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.(\forall V2c \in ty\_2Erat\_2Erat.(((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V0a)\ V1b))\land(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ V1b)\ V2c)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V0a)\ V2c)))))) \tag{119}$$

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.(\forall V2c \in ty\_2Erat\_2Erat.(((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ V0a)\ V1b))\land(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V1b)\ V2c)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V0a)\ V2c)))))) \tag{120}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V0r1))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_leq}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V1r2))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V1r2))))))) \tag{121}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.(\forall V2r3 \in ty\_2Erat\_2Erat.((V0r1 = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_sub}\ V1r2)\ V2r3))\Leftrightarrow((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V2r3) = V1r2))))) \tag{122}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.(\forall V2r3 \in ty\_2Erat\_2Erat.(((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V0r1)\ V2r3) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ V1r2)\ V2r3))\Leftrightarrow(V0r1 = V1r2))))) \tag{123}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ V1r2)\ V0r1))))) \tag{124}$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V0r1))\ V1r2))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ V1r2))\ V0r1))))) \tag{125}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0n \in A\_27a.(((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Erat\_2Erat\_\_0})\wedge(\forall V1n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V1n))\ \mathsf{c\_2Erat\_2Erat\_\_1}))))) \tag{126}$$

Assume the following.

$$(\mathsf{c\_2Erat\_2Erat\_\_0} = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})) \tag{127}$$

Assume the following.

$$(\mathsf{c\_2Erat\_2Erat\_\_1} = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \tag{128}$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat.((V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ c\_2Erat\_2ERATD\ V0r))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ c\_2Erat\_2ERATD\ V0r)))\wedge((((ap\ c\_2Erat\_2ERATN\ V0r) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\Rightarrow((ap\ c\_2Erat\_2ERATD\ V0r) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge(\forall V1n\_27 \in ty\_2Einteger\_2Eint.(\forall V2d\_27 \in ty\_2Enum\_2Enum.(((V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V1n\_27))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V2d\_27)))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2d\_27)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ \mathsf{c\_2Einteger\_2EABS}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Einteger\_2EABS}\ V1n\_27))))))))))) \tag{129}$$

Assume the following.

$$(\forall V0r \in ty\_2Erat\_2Erat.(V0r = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_div}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ (ap\ c\_2Erat\_2ERATN\ V0r)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ c\_2Erat\_2ERATD\ V0r))))) \tag{130}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V0n)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V0n)\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{131}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{132}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{133}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{134}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{135}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{136}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{137}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{138}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{139}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{140}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{141}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{142}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{143}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{144}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{145}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{146}$$

Theorem 1

$$(\forall V0r \in ty\_2Erat\_2Erat.((ap\ \mathsf{c\_2Erat\_2Erat\_\_sgn}\ V0r) = (ap\ \mathsf{c\_2EintExtension\_2ESGN}\ (ap\ c\_2Erat\_2ERATN\ V0r))))$$