

thm_2Erat_2ERAT__SGN__CALCULATE (TMN3PJZ6kLzdA3d8Ua6E3YeeK4UJg7ig3mc)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

Let `ty_2Efrac_2Efrac` : ι be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \tag{1}$$

Let `ty_2Erat_2Erat` : ι be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \tag{2}$$

Let `c_2Erat_2Erep__rat__CLASS` : ι be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \tag{3}$$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V 1x \in 2.V 1x)) (\lambda V 1x \in 2.V 1x)))$

Definition 5 We define `c_2Erat_2Erep__rat` to be $\lambda V 0a \in ty_2Erat_2Erat. (ap (c_2Emin_2E_40 ty_2Efrac_2Efrac))$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{4}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{5}$$

Let `c_2Efrac_2Erep__frac` : ι be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \tag{6}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (7)$$

Definition 6 We define $c_2Efrac_2Efrac_nrm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2EFST\ ty_2Efrac_2Efrac_nrm\ f))$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (12)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$.

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (15)$$

Definition 12 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (t$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod ty_2Enum_2Enum \\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \end{aligned} \quad (16)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (17)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})} \quad (18)$$

Definition 13 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 14 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (19)$$

Definition 15 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 16 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 18 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V0t))))$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(A_27a t1 t2))))$

Definition 20 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap (ap (ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D_3D_3E) (c_2Ebool_2E_2F_5C))))$

Definition 21 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2EintExtension_2ESGN (ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D_3D_3E) (c_2Ebool_2E_2F_5C))))$

Definition 22 We define $c_2Erat_2Erat_sgn$ to be $\lambda V0r \in ty_2Erat_2Erat.(ap c_2Efrac_2Efrac_sgn (ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D_3D_3E) (c_2Ebool_2E_2F_5C))))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \end{aligned} \quad (20)$$

Definition 23 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND (ap (c_2Ebool_2E_21 2) (c_2Emin_2E_3D_3D_3E) (c_2Ebool_2E_2F_5C))))$

