

thm_2Erat_2ERAT__SGN__NEG
(TMX4M4RpKdFQjvw3KAeHt9gyYkEdtZXRgEy)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define `c_2Earithmic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT1))$

Definition 8 We define `c_2Earithmic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (7)$$

Let `c_2Einteger_2Eint_of_num` : ι be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (8)$$

Let `ty_2Efrac_2Efrac` : ι be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (9)$$

Let `ty_2Erat_2Erat` : ι be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (10)$$

Let `c_2Erat_2Erep_rat_CLASS` : ι be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (11)$$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 10 We define `c_2Erat_2Erep_rat` to be $\lambda V0a \in ty_2Erat_2Erat.(ap (c_2Emin_2E_40 ty_2Efrac_2Efrac))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (12)$$

Let `c_2Efrac_2Erep_frac` : ι be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (13)$$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \quad (14)$$

Definition 11 We define `c_2Efrac_2Efrac_dnm` to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND ty_2Efrac_2Efrac))$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EFST\ A.27a\ A.27b \in (A.27a)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \quad (15)$$

Definition 12 We define $c_Efrac_Efrac_nmr$ to be $\lambda V0f \in ty_Efrac_Efrac.(ap (c_Epair_EFST ty_Eint_Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_Einteger_Eint_REP_CLASS \in ((2^{(ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)})^{ty_Einteger_Eint}) \quad (16)$$

Definition 13 We define $c_Einteger_Eint_REP$ to be $\lambda V0a \in ty_Einteger_Eint.(ap (c_Emin_E.40 (ty_Einteger_Eint_neg : \iota$ be given. Assume the following.

$$c_Einteger_Eint_neg \in ((ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)^{(ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)}) \quad (17)$$

Let $c_Einteger_Eint_eq : \iota$ be given. Assume the following.

$$c_Einteger_Eint_eq \in ((2^{(ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)})^{(ty_Epair_Eprod ty_Eenum_Eenum)}) \quad (18)$$

Let $c_Einteger_Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Einteger_Eint_ABS_CLASS \in (ty_Einteger_Eint)^{(2^{(ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)})} \quad (19)$$

Definition 14 We define $c_Einteger_Eint_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)$

Definition 15 We define $c_Einteger_Eint_neg$ to be $\lambda V0T1 \in ty_Einteger_Eint.(ap c_Einteger_Eint_neg$

Definition 16 We define $c_Emin_E.3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 17 We define $c_Ebool_E.2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E.21 2) (\lambda V2t \in 2.$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EABS_prod A_27a A_27b \in ((ty_Epair_Eprod A_27a A_27b)^{(2^{A_27b} A_27a})} \quad (20)$$

Definition 18 We define $c_Epair_E.2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_Epair_E.2C$

Let $c_Efrac_Eabs_frac : \iota$ be given. Assume the following.

$$c_Efrac_Eabs_frac \in (ty_Efrac_Efrac)^{(ty_Epair_Eprod ty_Einteger_Eint ty_Einteger_Eint)} \quad (21)$$

Definition 19 We define $c_Efrac_Efrac_ainv$ to be $\lambda V0f1 \in ty_Efrac_Efrac.(ap c_Efrac_Eabs_frac$

Let $c_Einteger_Eint_mul : \iota$ be given. Assume the following.

$$c_Einteger_Eint_mul \in (((ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)^{(ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)})^{(ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)}) \quad (22)$$

Definition 20 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$.
Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)^(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (23)$$

Definition 21 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$.

Definition 22 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$.

Definition 23 We define $c_2Efrac_2Efrac_sub$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$.

Definition 24 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$.

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (24)$$

Definition 25 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap c_2Erat_2Eabs_rat_CLASS)$.

Definition 26 We define $c_2Erat_2Erat_sub$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (25)$$

Definition 27 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$.

Definition 28 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Definition 30 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap (ap (ap (c_2Ebool$

Definition 31 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2EintExtension_2ESGN$

Definition 32 We define $c_2Erat_2Erat_sgn$ to be $\lambda V0r \in ty_2Erat_2Erat.(ap c_2Efrac_2Efrac_sgn (ap c_2Efrac$

Definition 33 We define $c_2Erat_2Erat_les$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 34 We define $c_2Erat_2Erat_gre$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 35 We define $c_2Efrac_2Efrac_1$ to be $(ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Efrac$

Definition 36 We define $c_2Erat_2Erat_1$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_1)$.

Definition 37 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 38 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 39 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 40 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Earithmetic_2Enum_CASE A_27a \in \left(((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum} \right) \quad (26)$$

Definition 41 We define $c_2Efrac_2Efrac_0$ to be $(ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum))))$

Definition 42 We define $c_2Erat_2Erat_0$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_0)$.

Definition 43 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 44 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40))))$

Definition 45 We define $c_2ERelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (27)$$

Definition 46 We define $c_2ERelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1g \in (A_27a^{A_27b}).$

Definition 47 We define $c_2ERelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b \in A_27a.$

Definition 48 We define $c_2ERelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).$

Definition 49 We define $c_2ERelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).$

Definition 50 We define $c_2ERelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M \in (A_27a^{A_27b}).$

Definition 51 We define $c_2Erat_2Erat_of_num$ to be $(ap (ap (c_2ERelation_2EWFREC ty_2Enum_2Enum)))$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0r1 \in ty_2Erat_2Erat.((((ap c_2Erat_2Erat_sgn V0r1) = \\ & (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num (\\ & ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Leftrightarrow \\ & (p (ap (ap c_2Erat_2Erat_les V0r1) (ap c_2Erat_2Erat_of_num \\ & c_2Enum_2E0)))) \wedge (((ap c_2Erat_2Erat_sgn V0r1) = (ap c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0)) \Leftrightarrow (V0r1 = (ap c_2Erat_2Erat_of_num c_2Enum_2E0))) \wedge \\ & (((ap c_2Erat_2Erat_sgn V0r1) = (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \Leftrightarrow \\ & (p (ap (ap c_2Erat_2Erat_gre V0r1) (ap c_2Erat_2Erat_of_num \\ & c_2Enum_2E0)))))) \end{aligned} \quad (30)$$

Theorem 1

$$\begin{aligned} & (\forall V0r \in ty_2Erat_2Erat.(((ap\ c_2Erat_2Erat_sgn\ V0r) = \\ & (ap\ c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ (\\ & ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \Leftrightarrow \\ & (p\ (ap\ (ap\ c_2Erat_2Erat_les\ V0r)\ (ap\ c_2Erat_2Erat_of_num \\ & c_2Enum_2E0)))) \end{aligned}$$