

thm_2Erat_2ERAT__SGN__POS (TMdK8w2pLrdb38itSnRBnBhh7TQtGcSifN9)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ c_2Enum_2EREP_num\ (c_2Enum_2ESUC_REP\ m)))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define `c_2Earithmetic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1) n)$

Definition 8 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (7)$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint)^{ty_2Enum_2Enum} \quad (8)$$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (9)$$

Let $ty_2Erat_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (10)$$

Let $c_2Erat_2Erep_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (11)$$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p\ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 10 We define `c_2Erat_2Erep_rat` to be $\lambda V0a \in ty_2Erat_2Erat.(ap (c_2Emin_2E_40\ ty_2Efrac_2Efrac) a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (12)$$

Let $c_2Efrac_2Erep_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (13)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (14)$$

Definition 11 We define `c_2Efrac_2Efrac_dnm` to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND\ ty_2Efrac_2Efrac) f)$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (15)$$

Definition 12 We define $c_Efrac_Efrac_nmr$ to be $\lambda V0f \in ty_Efrac_Efrac.(ap (c_Epair_EFST ty_Efrac_Efrac))$.
Let $c_Einteger_Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_Einteger_Eint_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})^{ty_Einteger_Eint})$$
(16)

Definition 13 We define $c_Einteger_Eint_REP$ to be $\lambda V0a \in ty_Einteger_Eint.(ap (c_Emin_E40 ty_Einteger_Eint))$.
Let $c_Einteger_Eint_neg : \iota$ be given. Assume the following.

$$c_Einteger_Eint_neg \in ((ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})$$
(17)

Let $c_Einteger_Eint_eq : \iota$ be given. Assume the following.

$$c_Einteger_Eint_eq \in ((2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})^{(ty_Epair_Eprod\ ty_Eenum_Eenum)})$$
(18)

Let $c_Einteger_Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Einteger_Eint_ABS_CLASS \in (ty_Einteger_Eint)^{(2^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})}$$
(19)

Definition 14 We define $c_Einteger_Eint_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)$.

Definition 15 We define $c_Einteger_Eint_neg$ to be $\lambda V0T1 \in ty_Einteger_Eint.(ap\ c_Einteger_Eint_neg)$.

Definition 16 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow Q)$ of type ι .

Definition 17 We define $c_Ebool_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21\ 2)) (\lambda V2t \in 2)))$.

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EABS_prod\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b} A_27a})})$$
(20)

Definition 18 We define c_Epair_E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_Epair_E2C))$.

Let $c_Efrac_Eabs_frac : \iota$ be given. Assume the following.

$$c_Efrac_Eabs_frac \in (ty_Efrac_Efrac)^{(ty_Epair_Eprod\ ty_Einteger_Eint\ ty_Einteger_Eint)}$$
(21)

Definition 19 We define $c_Efrac_Efrac_ainv$ to be $\lambda V0f1 \in ty_Efrac_Efrac.(ap\ c_Efrac_Eabs_frac)$.

Let $c_Einteger_Eint_mul : \iota$ be given. Assume the following.

$$c_Einteger_Eint_mul \in (((ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)})$$
(22)

Definition 39 We define $c_Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 40 We define $c_Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Earithmetic_2Enum_CASE A_27a \in ((A_27a^{(A_27a^{ty_2Enum_2Enum})})^{A_27a})^{ty_2Enum_2Enum} \quad (26)$$

Definition 41 We define $c_2Efrac_2Efrac_0$ to be $(ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum))))$

Definition 42 We define $c_2Erat_2Erat_0$ to be $(ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_0)$.

Definition 43 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 44 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_3D_3D_3E V0P) c_2Ebool_2E_7E)))$

Definition 45 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap (c_2Ebool_2E_2I V0R) c_2Ebool_2E_3F))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (27)$$

Definition 46 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1a \in A_27a. \lambda V2b \in A_27b. (ap (ap (ap (ap (c_2Erelation_2EWF A_27a V0f) V1a) V2b) c_2Ebool_2E_7E))$

Definition 47 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b \in A_27b. (ap (ap (ap (ap (c_2Erelation_2ERESTRICT A_27a V0R) V1a) V2b) c_2Ebool_2E_7E))$

Definition 48 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in A_27a. \lambda V2M \in A_27b. (ap (ap (ap (ap (c_2Erelation_2ETC A_27a V0R) V1M) V2M) c_2Ebool_2E_7E))$

Definition 49 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in A_27a. \lambda V2M \in A_27b. (ap (ap (ap (ap (c_2Erelation_2Eapprox A_27a V0R) V1M) V2M) c_2Ebool_2E_7E))$

Definition 50 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in A_27a. \lambda V2M \in A_27b. (ap (ap (ap (ap (c_2Erelation_2Ethe_fun A_27a V0R) V1M) V2M) c_2Ebool_2E_7E))$

Definition 51 We define $c_2Erat_2Erat_of_num$ to be $(ap (ap (c_2Erelation_2EWFREC ty_2Enum_2Enum)))$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0r1 \in ty_2Erat_2Erat. (((ap c_2Erat_2Erat_sgn V0r1) = \\ & (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num (\\ & ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Leftrightarrow \\ & (p (ap (ap c_2Erat_2Erat_les V0r1) (ap c_2Erat_2Erat_of_num \\ & c_2Enum_2E0)))) \wedge (((ap c_2Erat_2Erat_sgn V0r1) = (ap c_2Einteger_2Eint_of_num \\ & c_2Enum_2E0)) \Leftrightarrow (V0r1 = (ap c_2Erat_2Erat_of_num c_2Enum_2E0))) \wedge \\ & (((ap c_2Erat_2Erat_sgn V0r1) = (ap c_2Einteger_2Eint_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Leftrightarrow \\ & (p (ap (ap c_2Erat_2Erat_gre V0r1) (ap c_2Erat_2Erat_of_num \\ & c_2Enum_2E0)))))) \end{aligned} \quad (30)$$

Theorem 1

$$\begin{aligned} & (\forall V0r \in ty_2Erat_2Erat. (((ap\ c_2Erat_2Erat_sgn\ V0r) = \\ & (ap\ c_2Einteger_2Eint_of_num\ (ap\ c_2Earithmetric_2ENUMERAL \\ & (ap\ c_2Earithmetric_2EBIT1\ c_2Earithmetric_2EZERO)))) \Leftrightarrow (p\ (ap \\ & (ap\ c_2Erat_2Erat_les\ (ap\ c_2Erat_2Erat_of_num\ c_2Enum_2E0)) \\ & V0r)))) \end{aligned}$$