

thm\_2Erat\_2ERAT\_\_SUB\_\_CONG2  
(TMG83xbyu5MMCxkyxME56sr6aqRtQ4ZtRyd)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{3}$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{4}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2ESND\ ty\_2Efrac\_2Efrac\ f\ f))$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \tag{6}$$

**Definition 5** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2EFST ty\_2Efrac\_2Efrac))$ .  
Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (7)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}\ ty\_2Einteger\_2Eint)) \quad (8)$$

**Definition 6** We define  $c\_2Emin\_2E.40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap\ P\ x))$  **then**  $(the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E.40\ ty\_2Einteger\_2Eint))$ .

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (9)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (10)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})} \quad (11)$$

**Definition 8** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$ .

**Definition 9** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Eint\_neg)$ .

**Definition 10** We define  $c\_2Emin\_2E.3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E.2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E.21\ 2)) (\lambda V2t \in 2)))$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b}})^{A\_27a}) \quad (12)$$

**Definition 12** We define  $c\_2Epair\_2E.2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b))$ .

Let  $c\_2Efrac\_2Eabs\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Eabs\_frac \in (ty\_2Efrac\_2Efrac)^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)} \quad (13)$$

**Definition 13** We define  $c\_2Efrac\_2Efrac\_ainv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_fn$   
Let  $ty\_2Erat\_2Erat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \quad (14)$$

Let  $c\_2Erat\_2Erep\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erat\_2Erep\_rat\_CLASS \in ((2^{ty\_2Efrac\_2Efrac})^{ty\_2Erat\_2Erat}) \quad (15)$$

**Definition 14** We define  $c\_2Erat\_2Erep\_rat$  to be  $\lambda V0a \in ty\_2Erat\_2Erat.(ap\ (c\_2Emin\_2E40\ ty\_2Efrac$   
Let  $c\_2Einteger\_2Eint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)_{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}))_{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \quad (16)$$

**Definition 15** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$   
Let  $c\_2Einteger\_2Eint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)_{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}))_{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \quad (17)$$

**Definition 16** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 17** We define  $c\_2Efrac\_2Efrac\_add$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 18** We define  $c\_2Efrac\_2Efrac\_sub$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 19** We define  $c\_2Erat\_2Erat\_equiv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

Let  $c\_2Erat\_2Eabs\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erat\_2Eabs\_rat\_CLASS \in (ty\_2Erat\_2Erat)^{(2^{ty\_2Efrac\_2Efrac})} \quad (18)$$

**Definition 20** We define  $c\_2Erat\_2Eabs\_rat$  to be  $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_rat\_CLASS$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0f1 \in ty\_2Efrac\_2Efrac. (\forall V1f2 \in ty\_2Efrac\_2Efrac. ((ap\ c\_2Efrac\_2Efrac\_ainv\ (ap\ (ap\ c\_2Efrac\_2Efrac\_sub\ V1f2)\ V0f1)) = (ap\ (ap\ c\_2Efrac\_2Efrac\_sub\ V0f1)\ V1f2)))) \quad (23)$$

Assume the following.

$$(\forall V0x \in ty\_2Efrac\_2Efrac. ((ap\ c\_2Erat\_2Eabs\_rat\ (ap\ c\_2Efrac\_2Efrac\_ainv\ (ap\ c\_2Erat\_2Erep\_rat\ (ap\ c\_2Erat\_2Eabs\_rat\ V0x)))) = (ap\ c\_2Erat\_2Eabs\_rat\ (ap\ c\_2Efrac\_2Efrac\_ainv\ V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0x \in ty\_2Efrac\_2Efrac. (\forall V1y \in ty\_2Efrac\_2Efrac. ((ap\ c\_2Erat\_2Eabs\_rat\ (ap\ (ap\ c\_2Efrac\_2Efrac\_sub\ (ap\ c\_2Erat\_2Erep\_rat\ (ap\ c\_2Erat\_2Eabs\_rat\ V0x)))\ V1y)) = (ap\ c\_2Erat\_2Eabs\_rat\ (ap\ (ap\ c\_2Efrac\_2Efrac\_sub\ V0x)\ V1y)))))) \quad (25)$$

### Theorem 1

$$(\forall V0x \in ty\_2Efrac\_2Efrac. (\forall V1y \in ty\_2Efrac\_2Efrac. ((ap\ c\_2Erat\_2Eabs\_rat\ (ap\ (ap\ c\_2Efrac\_2Efrac\_sub\ V0x)\ (ap\ c\_2Erat\_2Erep\_rat\ (ap\ c\_2Erat\_2Eabs\_rat\ V1y)))) = (ap\ c\_2Erat\_2Eabs\_rat\ (ap\ (ap\ c\_2Efrac\_2Efrac\_sub\ V0x)\ V1y))))))$$