

thm\_2Erat\_2Erat\_ABS\_\_REP\_\_CLASS  
 (TMVWVwRmmn-  
 Fev2mwFXwXY5TZLHXGfRvtqpe)

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Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{3}$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{4}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2ESND\ A.27a\ A.27b \in (A.27b)^{(ty\_2Epair\_2Eprod\ A.27a\ A.27b)} \tag{5}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A-27a})))$

**Definition 4** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ ty\_2Efrac\_2Efrac\ f))$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (6)$$

**Definition 5** We define  $c\_2Efrac\_2Efrac\_nrmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))$ . Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (7)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}\ ty\_2Einteger\_2Eint\_REP\_CLASS) \quad (8)$$

**Definition 6** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p\ (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 7** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (c\_2Emin\_2E40\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))$ . Let  $c\_2Einteger\_2Eint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (9)$$

Let  $c\_2Einteger\_2Eint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}\ ty\_2Einteger\_2Eint\_eq) \quad (10)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (11)$$

**Definition 8** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$ .

**Definition 9** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint.$

**Definition 10** We define  $c\_2Erat\_2Erat\_equiv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac.$

**Definition 11** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E40\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))))$ .

Let  $ty\_2Erat\_2Erat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \quad (12)$$

Let  $c\_2Erat\_2Erep\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erat\_2Erep\_rat\_CLASS \in ((2^{ty\_2Efrac\_2Efrac}\ ty\_2Erat\_2Erat) \quad (13)$$

Let  $c\_2Erat\_2Eabs\_rat\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erat\_2Eabs\_rat\_CLASS \in (ty\_2Erat\_2Erat)^{(2^{ty\_2Efrac\_2Efrac}) \quad (14)$$

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

Assume the following.

$$\begin{aligned}
& ((\forall V0a \in ty\_2Erat\_2Erat.((ap c\_2Erat\_2Eabs\_rat\_CLASS \\
& (ap c\_2Erat\_2Erep\_rat\_CLASS V0a)) = V0a)) \wedge (\forall V1r \in (2^{ty\_2Efrac\_2Efrac}). \\
& ((p (ap (\lambda V2c \in (2^{ty\_2Efrac\_2Efrac}). (ap (c\_2Ebool\_2E\_3F ty\_2Efrac\_2Efrac) \\
& (\lambda V3r \in ty\_2Efrac\_2Efrac. (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap \\
& c\_2Erat\_2Erat\_equiv V3r) V3r)) (ap (ap (c\_2Emin\_2E\_3D (2^{ty\_2Efrac\_2Efrac}) \\
& V2c) (ap c\_2Erat\_2Erat\_equiv V3r)))))) V1r)) \Leftrightarrow ((ap c\_2Erat\_2Erep\_rat\_CLASS \\
& (ap c\_2Erat\_2Eabs\_rat\_CLASS V1r)) = V1r))))
\end{aligned} \tag{15}$$

**Theorem 1**

$$\begin{aligned}
& ((\forall V0a \in ty\_2Erat\_2Erat.((ap c\_2Erat\_2Eabs\_rat\_CLASS \\
& (ap c\_2Erat\_2Erep\_rat\_CLASS V0a)) = V0a)) \wedge (\forall V1c \in (2^{ty\_2Efrac\_2Efrac}). \\
& ((\exists V2r \in ty\_2Efrac\_2Efrac. ((p (ap (ap c\_2Erat\_2Erat\_equiv \\
& V2r) V2r)) \wedge (V1c = (ap c\_2Erat\_2Erat\_equiv V2r)))) \Leftrightarrow ((ap c\_2Erat\_2Erep\_rat\_CLASS \\
& (ap c\_2Erat\_2Eabs\_rat\_CLASS V1c)) = V1c))))
\end{aligned}$$