thm_2Erat_2Erat__of_int__ADD (TMMqehe4T8EKQfnZF2eeN1y4N1s662hNTCF)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A$.**if** $(\exists x \in A.p \ (ap \ P \ x))$ **then** $(the \ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o \ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ V0P \ (ap \ ($c_2Emin_2E_40 A

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty\_2Epair\_2Eprod$$
$$A0 \ A1)$$
$$\tag{2}$$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Einteger\_2Eint \tag{3}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2}$$
$$\tag{4}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap \ (ap \ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ (ap \ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 6 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap \ ($c_2Emin_2E_40 $(ty.$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)}) \tag{5}$$

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Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$

(6)

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}$$

(7)

Definition 7 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 8 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty}$$

(8)

Definition 9 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Definition 10 *We define* c_2Einteger_2Eint__sub *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2$

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$

(9)

Definition 11 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Definition 12 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$
of type ι.

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 15 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty}$$

(10)

Definition 16 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteg$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum})$$

(11)

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum})$$

(12)

2

Definition 17 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (13)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (14)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (15)$$

Definition 18 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 19 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 20 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 21 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 22 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (16)$$

Definition 23 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 24 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 25 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \qquad (17)$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \qquad (18)$$

Definition 26 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \qquad (19)$$

Definition 27 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Enumeral\_2EiSUB : \iota$ be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\text{-}2Enum\_2Enum})^2) \tag{20}$$

Let $ty\_2Erat\_2Erat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erat\_2Erat \tag{21}$$

Definition 28 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Definition 29 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 30 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Einteger\_2Eint\_of\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\text{-}2Enum\_2Enum}) \tag{22}$$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{23}$$

Let $c\_2Erat\_2Erep\_rat\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Erep\_rat\_CLASS \in ((2^{ty\text{-}2Efrac\_2Efrac})^{ty\text{-}2Erat\_2Erat}) \tag{24}$$

Definition 31 *We define* c_2Erat_2Erep_rat *to be* $\lambda V0a \in ty\_2Erat\_2Erat.(ap\ ($c_2Emin_2E_40 ty_2Efrac

Let $c\_2Efrac\_2Erep\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint \atop ty\_2Einteger\_2Eint)^{ty\text{-}2Efrac\_2Efrac}) \tag{25}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \atop A\_27a\ A\_27b \in (A\_27b^{(ty\text{-}2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{26}$$

Definition 32 *We define* c_2Efrac_2Efrac_dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ t$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \atop A\_27a\ A\_27b \in (A\_27a^{(ty\text{-}2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{27}$$

Definition 33 *We define* c_2Efrac_2Efrac_nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$
$$(28)$$

Definition 34 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)})$$
$$(29)$$

Definition 35 *We define* c_2Efrac_2Efrac__ainv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_fr$

Definition 36 *We define* c_2Efrac_2Efrac__add *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 37 *We define* c_2Efrac_2Efrac__sub *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Definition 38 *We define* c_2Erat_2Erat__equiv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Let $c\_2Erat\_2Eabs\_\_rat\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erat\_2Eabs\_\_rat\_\_CLASS \in (ty\_2Erat\_2Erat^{(2^{ty\_2Efrac\_2Efrac})}) \quad (30)$$

Definition 39 *We define* c_2Erat_2Eabs__rat *to be* $\lambda V0r \in ty\_2Efrac\_2Efrac.(ap\ c\_2Erat\_2Eabs\_\_rat\_\_Cl$

Definition 40 *We define* c_2Erat_2Erat__sub *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 41 *We define* c_2EintExtension_2ESGN *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Eboo$

Definition 42 *We define* c_2Efrac_2Efrac__sgn *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2EintExtension\_2ES$

Definition 43 *We define* c_2Erat_2Erat__sgn *to be* $\lambda V0r \in ty\_2Erat\_2Erat.(ap\ c\_2Efrac\_2Efrac\_\_sgn\ (ap\ c\_2$

Definition 44 *We define* c_2Erat_2Erat__les *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap\ ($

Definition 45 *We define* c_2Erat_2Erat__leq *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap\ ($

Definition 46 *We define* c_2Efrac_2Efrac__0 *to be* $(ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2$

Definition 47 *We define* c_2Erat_2Erat__0 *to be* $(ap\ c\_2Erat\_2Eabs\_\_rat\ c\_2Efrac\_2Efrac\_\_0)$.

Definition 48 *We define* c_2Efrac_2Efrac__1 *to be* $(ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2$

Definition 49 *We define* c_2Erat_2Erat__1 *to be* $(ap\ c\_2Erat\_2Eabs\_\_rat\ c\_2Efrac\_2Efrac\_\_1)$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(31)$$

Definition 50 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2$

Definition 51 *We define* c_2Erat_2Erat__add *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.\lambda V1r2 \in ty\_2Erat\_2Erat.(ap$

Definition 52 *We define* c_2Einteger_2ENum *to be* $\lambda V0i \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 ty_2En

Definition 53 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 54 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27}$

Definition 55 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap$ $(ap$ (c_2Ecombin_2ES A_27a $(A\_27a^{A\_27a})$ A

Let $c\_2Earithmetic\_2Enum\_\_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2Enum\_\_CASE\ A\_27a \in \\ (((A\_27a^{(A\_27a^{ty\_2Enum\_2Enum})})^{A\_27a})^{ty\_2Enum\_2Enum}) \tag{32}$$

Definition 56 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2E_21

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{33}$$

Definition 57 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 58 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 59 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 60 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 61 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 62 *We define* c_2Erat_2Erat__of__num *to be* $(ap$ $(ap$ (c_2Erelation_2EWFREC ty_2Enum_2Enum

Definition 63 *We define* c_2Erat_2Erat__ainv *to be* $\lambda V0r1 \in ty\_2Erat\_2Erat.(ap$ c_2Erat_2Eabs__rat $(ap$ c_2

Definition 64 *We define* c_2Erat_2Erat__of__int *to be* $\lambda V0i \in ty\_2Einteger\_2Eint.(ap$ $(ap$ $(ap$ (c_2Ebool_2E

Assume the following.

$$((ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO})) = \\ (ap\ \text{c\_2Enum\_2ESUC}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1} \\ \text{c\_2Earithmetic\_2EZERO}))))$$

$$\tag{34}$$

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Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ ($$
$$ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))))$$

$$(35)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V0n)))$$

$$(36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (p\ (ap\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))))$$

$$(37)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \wedge$$
$$(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m) \wedge ($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))$$
$$V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) =$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V0m)\ V1n)))))))))))$$

$$(38)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m)\ V1n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))) \Rightarrow (p\ ($$
$$ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p))))))$$

$$(39)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(V0m = V1n) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \wedge (p\ ($$
$$ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m))))))$$

$$(40)$$

7

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ \forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_2B}\ V0m)\ V1n))\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_2B}\\ V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))))))) \tag{41}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \tag{42}$$

Assume the following.

$$True \tag{43}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\forall V1x \in\\ A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in 2.((\exists V1x \in\\ A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Rightarrow(\neg(p\ V0t)))) \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t))\Rightarrow((p\ V0t)\Rightarrow False))) \tag{50}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow\\ (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge\\ (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{51}$$

8

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge$$
$$(((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee$$
$$(p\ V0t)) \Leftrightarrow (p\ V0t)))))))$$

(52)

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow$$
$$True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (($$
$$(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))))$$

(53)

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge$$
$$((\neg False) \Leftrightarrow True)))$$

(54)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow$$
$$True))$$

(55)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in$$
$$A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))$$

(56)

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg($$
$$p\ V0t)))))))$$

(57)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})$$
$$V0t1)\ V1t2) = V1t2))))$$

(58)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in$$
$$A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x))))))$$

(59)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in$$
$$A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x))))))$$

(60)

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Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee(\\ (p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C))))))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\vee(p\ V1B))\Leftrightarrow((p\ V1B)\vee \\ (p\ V0A))))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(\\ p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\Rightarrow(p\ V1B))\Leftrightarrow((\neg(p\ V0A))\vee \\ (p\ V1B))))) \quad (64)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Leftrightarrow((p\ V0t)\Leftrightarrow False))) \quad (65)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow \\ ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \quad (66)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\ 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow \quad (67)\\ (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27))))))))$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\ ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V0x)\ V1y))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le} \\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num \\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \\ V1y)))))$$

$$(68)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint. \\ (\forall V2z \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le} \\ V0x)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V2z)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le} \\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg} \\ V2z)))\ V1y)))))))$$

$$(69)$$

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Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_le} \ V0x) \ V1y)) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$(ap \ c\_2Einteger\_2Eint\_of\_num \ \mathsf{c\_2Enum\_2E0})) \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$V1y) \ (ap \ \mathsf{c\_2Einteger\_2Eint\_neg} \ V0x)))))))$$

$$\tag{70}$$

Assume the following.

$$(\forall V0c \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.$$
$$(\forall V2y \in ty\_2Einteger\_2Eint.((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$(ap \ c\_2Einteger\_2Eint\_of\_num \ \mathsf{c\_2Enum\_2E0})) \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$V0c) \ V1x))) \Rightarrow ((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_lt} \ V2y) \ (ap \ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$V1x))) \Rightarrow ((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_le} \ (ap \ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0})) \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add} \ (ap \ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$V0c)) \ V2y))) \Leftrightarrow False))))))$$

$$\tag{71}$$

Assume the following.

$$(\forall V0c \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.$$
$$(\forall V2y \in ty\_2Einteger\_2Eint.((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$(ap \ c\_2Einteger\_2Eint\_of\_num \ \mathsf{c\_2Enum\_2E0})) \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$V0c) \ V1x))) \Rightarrow ((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_lt} \ V1x) \ (ap \ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$V2y))) \Rightarrow (((ap \ c\_2Einteger\_2Eint\_of\_num \ \mathsf{c\_2Enum\_2E0}) = (ap \ ($$
$$ap \ \mathsf{c\_2Einteger\_2Eint\_add} \ (ap \ \mathsf{c\_2Einteger\_2Eint\_neg} \ V0c)) \ V2y)) \Leftrightarrow$$
$$False))))))$$

$$\tag{72}$$

Assume the following.

$$(\forall V0y \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.$$
$$((ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add} \ V1x) \ V0y) = (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$V0y) \ V1x))))$$

$$\tag{73}$$

Assume the following.

$$(\forall V0z \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(\forall V2x \in ty\_2Einteger\_2Eint.((ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$V2x) \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add} \ V1y) \ V0z)) = (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$(ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add} \ V2x) \ V1y)) \ V0z)))))$$

$$\tag{74}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$(ap \ c\_2Einteger\_2Eint\_of\_num \ \mathsf{c\_2Enum\_2E0})) \ V0x) = V0x))$$

$$\tag{75}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_add}$$
$$V0x) \ (ap \ c\_2Einteger\_2Eint\_of\_num \ \mathsf{c\_2Enum\_2E0})) = V0x))$$

$$\tag{76}$$

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Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \quad (77)$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO}))))\ V0x) = V0x))$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(\forall V2z \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}$$
$$V1y)\ V2z))))))$$
$$(78)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)$$
$$V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg} \quad (79)$$
$$V0x))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y)))))$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})))$$
$$(80)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)$$
$$V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg} \quad (81)$$
$$V0x))\ V1y))))$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)$$
$$V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$V1y)))))$$
$$(82)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x)) = V0x)) \quad (83)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((\neg(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0x)\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}$$
$$V1y)\ V0x)))))$$
$$(84)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le} \ V0x) \ V1y)) \Leftrightarrow ((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_lt}$$
$$V0x) \ V1y)) \lor (V0x = V1y))))) \tag{85}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(\forall V2z \in ty\_2Einteger\_2Eint.(((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$V0x) \ V1y)) \land (p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le} \ V1y) \ V2z))) \Rightarrow (p \ (ap$$
$$(ap \ \mathsf{c\_2Einteger\_2Eint\_\_le} \ V0x) \ V2z)))))) \tag{86}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le} \ V0x) \ V1y)) \land (p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le}$$
$$V1y) \ V0x))) \Leftrightarrow (V0x = V1y)))) \tag{87}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_lt}$$
$$(ap \ c\_2Einteger\_2Eint\_\_of\_\_num \ \mathsf{c\_2Enum\_2E0})) \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$V0x))) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_lt} \ V0x) \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))))) \tag{88}$$

Assume the following.

$$((ap \ \mathsf{c\_2Einteger\_2Eint\_\_neg} \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) = (ap \ c\_2Einteger\_2Eint\_\_of\_\_num \ \mathsf{c\_2Enum\_2E0})) \tag{89}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le} \ V1y) \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_add}$$
$$V0x) \ V1y))) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le} \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) \ V0x))))) \tag{90}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_\_le} \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V0m)) \ (ap \ c\_2Einteger\_2Eint\_\_of\_\_num \ V1n))) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V0m) \ V1n))))) \tag{91}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap \ c\_2Einteger\_2Eint\_\_of\_\_num \ V0m) = (ap \ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V1n)) \Leftrightarrow (V0m = V1n)))) \tag{92}$$

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Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\\
V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))$$

(93)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.\\
((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\\
V0x))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\\
V1y)\ V0x)))))$$

(94)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))\Rightarrow((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_sub}\\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\\
V1m)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\\
V0n)\ V1m)))))))$$

(95)

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.\\
(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\\
V1y))\Leftrightarrow(V0x = V1y))))$$

(96)

Assume the following.

$$(\forall V0i \in ty\_2Einteger\_2Eint.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\\
(ap\ \mathsf{c\_2Einteger\_2ENum}\ V0i)) = V0i)\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0i))))$$

(97)

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1n \in ty\_2Enum\_2Enum.$$
$$(\forall V2m \in ty\_2Enum\_2Enum.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0p) = V0p) \wedge (((($$
$$ap\ (ap\ c\_2Einteger\_2Eint\_\_add\ V0p)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) = V0p) \wedge (((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \wedge$$
$$(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0p)) =$$
$$V0p) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m))) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V2m))))) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V2m)))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap$$
$$(ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V2m)\ V1n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n)$$
$$V2m))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2m)$$
$$V1n)))))) \wedge (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1n))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V2m))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2m)$$
$$V1n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n)$$
$$V2m)))))) \wedge ((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m)))) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2m))))))))))))))))))$$
$$\tag{98}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$\mathsf{c\_2Enum\_2E0})\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\Leftrightarrow$$
$$True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$\mathsf{c\_2Enum\_2E0})\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$\mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap}$$
$$\mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap}$$
$$c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap}$$
$$\mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$V1m))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow False)\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}}$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}}$$
$$V1m)))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n)))))))))))))))))$$

$$(99)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow$$
$$True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))))$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge(((p$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge(((p$$
$$(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap$$
$$c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap$$
$$\mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap$$
$$\mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))))))\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap$$
$$c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ ($$
$$ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))))))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))))))\Leftrightarrow$$
$$False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\ (ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V1m))))\Leftrightarrow True)\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_le}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_of\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V0n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_neg}\ (ap\ c\_2Einteger\_2Eint\_of\_num}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V1m)\ V0n)))))))))))))))))))$$

$$(100)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$\forall V2x \in ty\_2Einteger\_2Eint.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V1m)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0n)\ V1m)))\wedge(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V1m)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap\ (ap$$
$$\mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1m)\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V0n)\ V1m)))))\wedge(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V0n))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V1m))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V1m)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V1m)\ V0n)))))\wedge(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m))) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0n)\ V1m))))\wedge(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V0n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0n)\ V1m)))\wedge(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)))$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V0n)\ V1m))))\wedge(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V0n))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V1m))) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0n)\ V1m))))\wedge(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)))$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m))) =$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V0n)\ V1m)))\wedge((((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V1m))\Leftrightarrow(V0n = V1m))\wedge((((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n) = ($$
$$ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\Leftrightarrow$$
$$((V0n = \mathsf{c\_2Enum\_2E0})\wedge(V1m = \mathsf{c\_2Enum\_2E0})))\wedge((((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$V1m))\Leftrightarrow((V0n = \mathsf{c\_2Enum\_2E0})\wedge(V1m = \mathsf{c\_2Enum\_2E0})))\wedge((((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0n)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1m)))\Leftrightarrow(V0n = V1m))\wedge(((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V2x)) = V2x)\wedge((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}$$
$$(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))))))))))))))))))$$

$$(101)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0})))$$
$$(102)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0}))\wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n))\Rightarrow(p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))$$
$$(103)$$

Assume the following.

$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$\textsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \textsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in$
$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V3m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Enumeral\_2EiZ}\ (ap$
$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \textsf{c\_2Enum\_2E0})\ V4n) = \textsf{c\_2Enum\_2E0}))\wedge$
$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V5n)\ \textsf{c\_2Enum\_2E0}) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.$
$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($
$ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V7m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$
$V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$\textsf{c\_2Enum\_2E0})\ V8n) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \textsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in$
$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V11m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$
$V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$
$\textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$
$V12n))) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap$
$(ap\ c\_2Earithmetic\_2EEXP\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \textsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in$
$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \textsf{c\_2Enum\_2E0}) =$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}\ \textsf{c\_2Earithmetic\_2EZERO}))))\wedge$
$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$
$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \textsf{c\_2Enum\_2ESUC}$
$\textsf{c\_2Enum\_2E0}) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \textsf{c\_2Earithmetic\_2EBIT1}$
$\textsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.($
$(ap\ \textsf{c\_2Enum\_2ESUC}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ \textsf{c\_2Enum\_2E0}) =$
$\textsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}$
$(ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$(ap\ \textsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.$
$(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \textsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \textsf{c\_2Earithmetic\_2EZERO})))\wedge$
$((\forall V20n \in ty\_2Enum\_2Enum.((\textsf{c\_2Enum\_2E0} = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V20n))\Leftrightarrow(V20n = \textsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.$
$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V21n) = (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge$
$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V23n)\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Enum\_2E0})\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$
$V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V25n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}$
$\textsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V28n))\ \textsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}\ \textsf{c\_2Earithmetic\_2EZERO})$
$V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$
$((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}$
$V29n))\ (ap\ \textsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \textsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \textsf{c\_2Earithmetic\_2E\_3C\_3D}$
$\textsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$

$((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ \mathsf{c\_2Earithmetic\_2EZERO}) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$
$V0n))) {\Leftrightarrow} True) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ \mathsf{c\_2Earithmetic\_2EZERO})$
$(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V0n))) {\Leftrightarrow} True) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V0n) \ \mathsf{c\_2Earithmetic\_2EZERO})) {\Leftrightarrow} False) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$(ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) {\Leftrightarrow}$
$(p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ V0n) \ V1m))) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) {\Leftrightarrow}$
$(p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ V0n) \ V1m))) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$(ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) {\Leftrightarrow}$
$({\neg}(p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ V1m) \ V0n)))) {\wedge} ((p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) {\Leftrightarrow}$
$(p \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \ V0n) \ V1m))))))))))))$

(105)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$

$((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ \mathsf{c\_2Earithmetic\_2EZERO}) \ V0n)) {\Leftrightarrow}$
$True) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$
$V0n)) \ \mathsf{c\_2Earithmetic\_2EZERO})) {\Leftrightarrow} False) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$
$(ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V0n)) \ \mathsf{c\_2Earithmetic\_2EZERO})) {\Leftrightarrow} False) {\wedge}$
$(((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$
$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) {\Leftrightarrow} (p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$
$V0n) \ V1m))) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$
$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) {\Leftrightarrow} (p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$
$V0n) \ V1m))) {\wedge} (((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$
$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ V1m))) {\Leftrightarrow} ({\neg}(p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$
$V1m) \ V0n)))) {\wedge} ((p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2}$
$V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT2} \ V1m))) {\Leftrightarrow} (p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$
$V0n) \ V1m))))))))))))$

(106)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$

$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2D \ V0n)$
$V1m)) = (ap \ (ap \ (ap \ (\mathsf{c\_2Ebool\_2ECOND} \ ty\_2Enum\_2Enum) \ (ap \ (ap \ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$
$V1m) \ V0n)) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ (ap \ (ap \ c\_2Enumeral\_2EiSUB$
$\mathsf{c\_2Ebool\_2ET}) \ V0n) \ V1m))) \ \mathsf{c\_2Enum\_2E0})))$

(107)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$

$((ap \ \mathsf{c\_2Enum\_2ESUC} \ V0m) = (ap \ \mathsf{c\_2Enum\_2ESUC} \ V1n)) {\Leftrightarrow} (V0m = V1n))))$

(108)

Assume the following.

$$(\forall V0a \in ty\_2Erat\_2Erat.(\forall V1b \in ty\_2Erat\_2Erat.($$
$$(ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_add} \ V0a) \ V1b) = (ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_add} \quad (109)$$
$$V1b) \ V0a))))$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$((ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_sub} \ V0r1) \ V1r2) = (ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_add}$$
$$V0r1) \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_ainv} \ V1r2)))))$$

$$(110)$$

Assume the following.

$$((ap \ \mathsf{c\_2Erat\_2Erat\_\_ainv} \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num} \ \mathsf{c\_2Enum\_2E0})) =$$
$$(ap \ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num} \ \mathsf{c\_2Enum\_2E0})) \quad (111)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$((ap \ \mathsf{c\_2Erat\_2Erat\_\_ainv} \ (ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_add} \ V0r1) \ V1r2)) =$$
$$(ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_add} \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_ainv} \ V0r1)) \ (ap \quad (112)$$
$$\mathsf{c\_2Erat\_2Erat\_\_ainv} \ V1r2)))))$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$(((ap \ \mathsf{c\_2Erat\_2Erat\_\_ainv} \ V0r1) = V1r2) \Leftrightarrow (V0r1 = (ap \ \mathsf{c\_2Erat\_2Erat\_\_ainv}$$
$$V1r2)))))$$

$$(113)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$(((ap \ \mathsf{c\_2Erat\_2Erat\_\_ainv} \ V0r1) = (ap \ \mathsf{c\_2Erat\_2Erat\_\_ainv} \ V1r2)) \Leftrightarrow \quad (114)$$
$$(V0r1 = V1r2))))$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$((p \ (ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_les} \ V0r1) \ V1r2)) \Rightarrow (\neg (p \ (ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_les}$$
$$V1r2) \ V0r1))))))$$

$$(115)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(p \ (ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_leq} \ V0r1)$$
$$V0r1))) \quad (116)$$

Assume the following.

$$(p \ (ap \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_les} \ (ap \ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num} \ \mathsf{c\_2Enum\_2E0}))$$
$$(ap \ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num} \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap$$
$$\mathsf{c\_2Earithmetic\_2EBIT1} \ \mathsf{c\_2Earithmetic\_2EZERO})))))$$

$$(117)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_of\_num}\ \mathsf{c\_2Enum\_2E0}))$$
$$V0r1))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_leq}\ (ap\ \mathsf{c\_2Erat\_2Erat\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0}))\ V1r2))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_add}\ V0r1)\ V1r2)))))))$$

$$(118)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$(\forall V2r3 \in ty\_2Erat\_2Erat.((V0r1 = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_sub}$$
$$V1r2)\ V2r3))\Leftrightarrow((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_add}\ V0r1)\ V2r3) = V1r2)))))$$

$$(119)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$(\forall V2r3 \in ty\_2Erat\_2Erat.(((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_add}\ V0r1)$$
$$V2r3) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_add}\ V1r2)\ V2r3))\Leftrightarrow(V0r1 = V1r2)))))$$

$$(120)$$

Assume the following.

$$(\forall V0r1 \in ty\_2Erat\_2Erat.(\forall V1r2 \in ty\_2Erat\_2Erat.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_les}\ (ap\ \mathsf{c\_2Erat\_2Erat\_ainv}\ V0r1))$$
$$(ap\ \mathsf{c\_2Erat\_2Erat\_ainv}\ V1r2)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_les}$$
$$V1r2)\ V0r1)))))$$

$$(121)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0n \in A\_27a.(((ap\ \mathsf{c\_2Erat\_2Erat\_of\_num}$$
$$\mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Erat\_2Erat\_0})\wedge(\forall V1n \in ty\_2Enum\_2Enum.$$
$$((ap\ \mathsf{c\_2Erat\_2Erat\_of\_num}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_add}$$
$$(ap\ \mathsf{c\_2Erat\_2Erat\_of\_num}\ V1n))\ \mathsf{c\_2Erat\_2Erat\_1})))))$$

$$(122)$$

Assume the following.

$$(\mathsf{c\_2Erat\_2Erat\_0} = (ap\ \mathsf{c\_2Erat\_2Erat\_of\_num}\ \mathsf{c\_2Enum\_2E0}))\qquad(123)$$

Assume the following.

$$(\mathsf{c\_2Erat\_2Erat\_1} = (ap\ \mathsf{c\_2Erat\_2Erat\_of\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))$$

$$(124)$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V0n))$$
$$(ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V1m)) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))))\wedge((\forall V2n \in ty\_2Enum\_2Enum.$$
$$(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap$$
$$\mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V2n)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}$$
$$V3m)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erat\_2Erat)\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V2n)\ V3m))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V3m)\ V2n)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2n)\ V3m)))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.$$
$$(\forall V5m \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap$$
$$\mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V4n)\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}$$
$$V5m))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Erat\_2Erat)\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$V5m)\ V4n))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V4n)\ V5m)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V5m)\ V4n)))))))\wedge(\forall V6n \in ty\_2Enum\_2Enum.$$
$$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap$$
$$\mathsf{c\_2Erat\_2Erat\_\_ainv}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V6n)))\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}$$
$$(ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ V7m))) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_ainv}\ ($$
$$ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_num}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V6n)\ V7m)))))))))$$

$$(125)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{126}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{127}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow$$
$$(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{128}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow$$
$$((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{129}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{130}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg($$
$$p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee$$
$$((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))))$$

$$(131)$$

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Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}(\\
(p\ V1q){\wedge}(p\ V2r)))){\Leftrightarrow}(((p\ V0p){\vee}((\neg(p\ V1q)){\vee}(\neg(p\ V2r)))){\wedge}(((p\ V1q){\vee}\\
(\neg(p\ V0p))){\wedge}((p\ V2r){\vee}(\neg(p\ V0p))))))))))$$

(132)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}(\\
(p\ V1q){\vee}(p\ V2r))){\Leftrightarrow}(((p\ V0p){\vee}(\neg(p\ V1q))){\wedge}(((p\ V0p){\vee}(\neg(p\ V2r))){\wedge}\\
((p\ V1q){\vee}((p\ V2r){\vee}(\neg(p\ V0p)))))))))))$$

(133)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}(\\
(p\ V1q){\Rightarrow}(p\ V2r))){\Leftrightarrow}(((p\ V0p){\vee}(p\ V1q)){\wedge}(((p\ V0p){\vee}(\neg(p\ V2r))){\wedge}((\\
\neg(p\ V1q)){\vee}((p\ V2r){\vee}(\neg(p\ V0p)))))))))))$$

(134)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p){\Leftrightarrow}(\neg(p\ V1q))){\Leftrightarrow}(((p\ V0p){\vee}\\
(p\ V1q)){\wedge}((\neg(p\ V1q)){\vee}(\neg(p\ V0p)))))))$$

(135)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p){\Rightarrow}(p\ V1q))){\Rightarrow}(p\ V0p))))$$

(136)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p){\Rightarrow}(p\ V1q))){\Rightarrow}(\neg(p\ V1q)))))$$

(137)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p){\vee}(p\ V1q))){\Rightarrow}(\neg(p\ V0p)))))$$

(138)

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p){\vee}(p\ V1q))){\Rightarrow}(\neg(p\ V1q)))))$$

(139)

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p))){\Rightarrow}(p\ V0p)))$$

(140)

Theorem 1

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.\\
((ap\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_add}\ (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V0x))\\
(ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\ V1y)) = (ap\ \mathsf{c\_2Erat\_2Erat\_\_of\_\_int}\\
(ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V1y)))))$$

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