

thm_2Erat_2Erat_of_int_LT
(TMYHytkVCJjnTrXro6SfgTqMgHfXh4L1h8T)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A \ V0P))$

Definition 4 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap (c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2EBOUNDED` to be $(\lambda V0v \in 2. \text{c_2Ebool_2E_2T})$.

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty_2Epair_2Eprod } A0 \ A1) \tag{2}$$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$\text{nonempty ty_2Einteger_2Eint} \tag{3}$$

Let `c_2Einteger_2Eint_REP_CLASS` : ι be given. Assume the following.

$$\text{c_2Einteger_2Eint_REP_CLASS} \in ((2^{(\text{ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum}) \text{ty_2Einteger_2Eint}})) \tag{4}$$

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap (c_2Emin_2E_3D } (2^{A-27a}) \ V0P))$

Definition 7 We define `c_2Einteger_2Eint_REP` to be $\lambda V0a \in \text{ty_2Einteger_2Eint}. (\text{ap (c_2Emin_2E_40 (ty_2Einteger_2Eint_REP_CLASS } V0a))$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum)\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum) \quad (5)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) \quad (6)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))} \quad (7)$$

Definition 8 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 9 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (9)$$

Definition 10 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21))$

Definition 13 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num)$

Definition 15 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21) 2) (\lambda V2t \in 2$

Definition 18 We define $c_Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{13}$$

Definition 19 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 20 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Definition 21 We define $c_Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{14}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{15}$$

Definition 22 We define $c_2Enumeral_2EiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{16}$$

Definition 23 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 24 We define $c_Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 25 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2)^{ty_2Enum_2Enum} \tag{17}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{18}$$

Definition 26 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 27 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 28 We define `c_2Earithmic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `c_2Einteger_2Eint_of_num` : ι be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (19)$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (20)$$

Definition 29 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2E_2C\ ty_2Efrac_2Efrac\ x\ y))$

Let `ty_2Efrac_2Efrac` : ι be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (21)$$

Let `c_2Efrac_2Eabs_frac` : ι be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (22)$$

Definition 30 We define `c_2Efrac_2Efrac_0` to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Efrac_2Efrac\ x\ y))\ z))$

Let `ty_2Erat_2Erat` : ι be given. Assume the following.

$$nonempty\ ty_2Erat_2Erat \quad (23)$$

Let `c_2Erat_2Erep_rat_CLASS` : ι be given. Assume the following.

$$c_2Erat_2Erep_rat_CLASS \in ((2^{ty_2Efrac_2Efrac})^{ty_2Erat_2Erat}) \quad (24)$$

Definition 31 We define `c_2Erat_2Erep_rat` to be $\lambda V0a \in ty_2Erat_2Erat.(ap\ (c_2Emin_2E_40\ ty_2Efrac_2Efrac\ a))$

Let `c_2Efrac_2Erep_frac` : ι be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (25)$$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (26)$$

Definition 32 We define `c_2Efrac_2Efrac_dnm` to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap\ (c_2Epair_2ESND\ ty_2Efrac_2Efrac\ f))$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (27)$$

Definition 33 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2Efst ty_2Eint_2Eint_neg))$

Let $c_2Einteger_2Eint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_neg \in ((ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)) \quad (28)$$

Definition 34 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint_neg)$

Definition 35 We define $c_2Efrac_2Efrac_ainv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2Efrac_2Eabs_fn)$

Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)))(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum) \quad (29)$$

Definition 36 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Let $c_2Einteger_2Eint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_lt \in ((2^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)})(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)) \quad (30)$$

Definition 37 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$

Definition 38 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$

Definition 39 We define $c_2Erat_2Erat_equiv$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Let $c_2Erat_2Eabs_rat_CLASS : \iota$ be given. Assume the following.

$$c_2Erat_2Eabs_rat_CLASS \in (ty_2Erat_2Erat^{(2^{ty_2Efrac_2Efrac})}) \quad (31)$$

Definition 40 We define $c_2Erat_2Eabs_rat$ to be $\lambda V0r \in ty_2Efrac_2Efrac.(ap c_2Erat_2Eabs_rat_CLASS)$

Definition 41 We define $c_2Efrac_2Efrac_add$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 42 We define $c_2Efrac_2Efrac_sub$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$

Definition 43 We define $c_2Erat_2Erat_sub$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap c_2Erat_2Eabs_rat)$

Definition 44 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap (ap (ap (c_2Eboo)))$

Definition 45 We define $c_2Efrac_2Efrac_sgn$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.(ap c_2EintExtension_2ESGN)$

Definition 46 We define $c_2Erat_2Erat_sgn$ to be $\lambda V0r \in ty_2Erat_2Erat.(ap c_2Efrac_2Efrac_sgn (ap c_2Erat_2Eabs_rat))$

Definition 47 We define $c_2Erat_2Erat_les$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap c_2Erat_2Erat_sgn)$

Definition 48 We define $c_2Erat_2Erat_leq$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap c_2Erat_2Erat_les)$

Definition 49 We define $c_2Erat_2Erat_add$ to be $\lambda V0r1 \in ty_2Erat_2Erat.\lambda V1r2 \in ty_2Erat_2Erat.(ap$

Definition 50 We define $c_2Erat_2Erat_0$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_0)$.

Definition 51 We define $c_2Efrac_2Efrac_1$ to be $(ap\ c_2Efrac_2Eabs_frac\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2E$

Definition 52 We define $c_2Erat_2Erat_1$ to be $(ap\ c_2Erat_2Eabs_rat\ c_2Efrac_2Efrac_1)$.

Definition 53 We define $c_2Einteger_2ENum$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap\ (c_2Emin_2E_40\ ty_2E$

Definition 54 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 55 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 56 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Let $c_2Earithmetic_2Enum_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in \quad (32)$$

$$(((A_27a^{(A_27a^{ty_2Enum_2Enum})})_{A_27a})_{ty_2Enum_2Enum})$$

Definition 57 We define $c_2ERelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap\ (c_2Ebool_2E_21$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (33)$$

Definition 58 We define $c_2ERelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1$

Definition 59 We define $c_2ERelation_2ETC$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Definition 60 We define $c_2ERelation_2Eapprox$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 61 We define $c_2ERelation_2Ethe_fun$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 62 We define $c_2ERelation_2EWFREC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 63 We define $c_2Erat_2Erat_of_num$ to be $(ap\ (ap\ (c_2ERelation_2EWFREC\ ty_2Enum_2Enum$

Definition 64 We define $c_2Erat_2Erat_ainv$ to be $\lambda V0r1 \in ty_2Erat_2Erat.(ap\ c_2Erat_2Eabs_rat\ (ap\ c_2$

Definition 65 We define $c_2Erat_2Erat_of_int$ to be $\lambda V0i \in ty_2Einteger_2Eint.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Assume the following.

$$((ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)) =$$

$$(ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1$$

$$c_2Earithmetic_2EZERO)))) \quad (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\
& \quad ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\
& \quad V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\
& \quad V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((\neg(p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad c_2Enum_2E0) V0n))) \Leftrightarrow (V0n = c_2Enum_2E0))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V0n))) \\
& \hspace{15em} (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad V1n) V0m)))))) \\
& \hspace{15em} (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& \quad (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& \quad (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& \quad V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& \quad V0m) V1n)))))))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))) \Rightarrow (p (\\
& \quad ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (41)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))) \quad (43)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m)))))) \quad (44)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0n))) \quad (45)$$

Assume the following.

$$True \quad (46)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (49)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (58)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in \\ A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in \\ A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p\ V0A) \Rightarrow (p\ V1B))) \Leftrightarrow ((p\ V0A) \wedge \\ (\neg(p\ V1B)))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ p\ V0A) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B))))))))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \vee \\ (p\ V1B)))))) \quad (68)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (69)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (70)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \Rightarrow (71)$$

Assume the following.

$$(\forall V0v \in 2.((p (ap c_2Ebool_2EBOUNDED V0v)) \Leftrightarrow True)) \quad (72)$$

Assume the following.

$$(\forall V0a \in ty_2Einteger_2Eint.(\forall V1b \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V1b)) \Rightarrow ((ap c_2Efrac_2Efrac_nmr (ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) V0a) V1b))) = V0a)))))) \quad (73)$$

Assume the following.

$$(\forall V0a \in ty_2Einteger_2Eint.(\forall V1b \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V1b)) \Rightarrow ((ap c_2Efrac_2Efrac_dnm (ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) V0a) V1b))) = V1b)))))) \quad (74)$$

Assume the following.

$$((ap c_2Efrac_2Efrac_ainv c_2Efrac_2Efrac_0) = c_2Efrac_2Efrac_0) \quad (75)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V1y)))))) \quad (76)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. (\forall V2z \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_le V0x) (ap (ap c_2Einteger_2Eint_add V1y) V2z))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_neg V2z))) V1y)))))) \quad (77)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((p (ap (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow (p (ap (ap (ap c_2Einteger_2Eint_le \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add \\
& V1y) (ap c_2Einteger_2Eint_neg V0x)))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. \\
& (\forall V2y \in ty_2Einteger_2Eint. ((p (ap (ap (ap c_2Einteger_2Eint_le \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add \\
& V0c) V1x))) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_lt V2y) (ap c_2Einteger_2Eint_neg \\
& V1x))) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& V0c) V2y))) \Leftrightarrow False))))))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Einteger_2Eint. (\forall V1x \in ty_2Einteger_2Eint. \\
& ((ap (ap c_2Einteger_2Eint_add V1x) V0y) = (ap (ap c_2Einteger_2Eint_add \\
& V0y) V1x))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\
& V2x) (ap (ap c_2Einteger_2Eint_add V1y) V0z)) = (ap (ap c_2Einteger_2Eint_add \\
& (ap (ap c_2Einteger_2Eint_add V2x) V1y)) V0z))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0x) = V0x))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\
& V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& V0x) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) = V0x))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap (ap c_2Einteger_2Eint_add V0x) V1y)) V2z) = (ap (ap c_2Einteger_2Eint_add \\
& (ap (ap c_2Einteger_2Eint_mul V0x) V2z)) (ap (ap c_2Einteger_2Eint_mul \\
& V1y) V2z))))))
\end{aligned} \tag{86}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_add V0x) \\
& V1y)) = (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& V0x)) (ap c_2Einteger_2Eint_neg V1y))))))
\end{aligned} \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0x) = (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
& V1y)) = (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg \\
& V0x)) V1y))))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
& V1y)) = (ap (ap c_2Einteger_2Eint_mul V0x) (ap c_2Einteger_2Eint_neg \\
& V1y))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_neg V0x)) = V0x))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((\neg(p (ap (ap c_2Einteger_2Eint_le V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt \\
& V1y) V0x))))))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2z \in ty_2Einteger_2Eint. (((p (ap (ap c_2Einteger_2Eint_le \\
& V0x) V1y)) \wedge (p (ap (ap c_2Einteger_2Eint_le V1y) V2z))) \Rightarrow (p (ap \\
& (ap c_2Einteger_2Eint_le V0x) V2z))))))
\end{aligned} \tag{93}$$

Assume the following.

$$(p (ap (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) (94)$$

Assume the following.

$$((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (95)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap (ap c_2Einteger_2Eint_le V1y) (ap (ap c_2Einteger_2Eint_add V0x) V1y))) \Leftrightarrow (p (ap (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0x)))))) (96)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num V0m)) (ap c_2Einteger_2Eint_of_num V1n))) \Leftrightarrow (p (ap (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)))))) (97)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num V0m)) (ap c_2Einteger_2Eint_of_num V1n))) \Leftrightarrow (p (ap (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))))) (98)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_neg V0x)) (ap c_2Einteger_2Eint_neg V1y))) \Leftrightarrow (p (ap (ap (ap c_2Einteger_2Eint_le V1y) V0x)))))) (99)$$

Assume the following.

$$(\forall V0i \in ty_2Einteger_2Eint. (((ap c_2Einteger_2Eint_of_num (ap c_2Einteger_2Enum V0i)) = V0i) \Leftrightarrow (p (ap (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0i)))))) (100)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2m \in ty_2Enum_2Enum. (((ap (ap c_2Einteger_2Eint_add \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0p) = V0p) \wedge (((\\
& ap (ap c_2Einteger_2Eint_add V0p) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = V0p) \wedge (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \wedge \\
& (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0p) = \\
& V0p) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m))) = (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& V1n) V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m)))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap \\
& (ap c_2Earithmetic_2E_3C_3D V2m) V1n)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap (\\
& ap c_2Earithmetic_2E_3C_3D V1n) V2m)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m)))) = (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V1n) V2m))))))))))))))
\end{aligned}$$

(101)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& \quad ((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad (ap c_2Arithmetic_2EBIT1 V0n)))))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& \quad (ap c_2Integer_2Eint_of_num c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num \\
& \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V0n)))))) \Leftrightarrow \\
& \quad True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad (ap c_2Arithmetic_2ENUMERAL V0n)))))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad V0n)) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge \\
& \quad (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad (ap c_2Arithmetic_2EBIT1 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap \\
& \quad c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad (ap c_2Arithmetic_2EBIT2 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap \\
& \quad c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL V0n))) \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap \\
& \quad c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT1 V0n)))))) \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad (ap c_2Arithmetic_2EBIT2 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& \quad (ap c_2Arithmetic_2ENUMERAL V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad V0n)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad (ap c_2Arithmetic_2ENUMERAL V1m)))))) \Leftrightarrow False) \wedge ((p (ap (ap c_2Integer_2Eint_lt \\
& \quad (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (\\
& \quad ap c_2Arithmetic_2ENUMERAL V0n)))) (ap c_2Integer_2Eint_neg \\
& \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad V1m)))))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))))))))) \\
& \quad (102)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& \quad ((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow \\
& \quad True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT1 V0n)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V0n)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT1 V0n)))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge (((p \\
& \quad \quad (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V0n)))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge (((p \\
& \quad \quad (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg (ap \\
& \quad \quad c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL (ap \\
& \quad \quad c_2Arithmetic_2EBIT1 V0n)))) (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap \\
& \quad \quad c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Arithmetic_2EBIT2 V0n)))) (ap c_2Integer_2Eint_of_num \\
& \quad \quad c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap \\
& \quad \quad c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL V0n))) \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V1m)))) \Leftrightarrow (p (ap (ap c_2Arithmetic_2E_3C_3D V0n) V1m))) \wedge (((p (\\
& \quad \quad ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL V0n))) (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Arithmetic_2EBIT1 V1m)))))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Integer_2Eint_le \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n))) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL (ap c_2Arithmetic_2EBIT2 V1m)))))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_le (ap c_2Integer_2Eint_neg \\
& \quad \quad (ap c_2Integer_2Eint_of_num (ap c_2Arithmetic_2ENUMERAL \\
& \quad \quad V0n)))) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& \quad \quad (ap c_2Arithmetic_2ENUMERAL V1m)))))) \Leftrightarrow (p (ap (ap c_2Arithmetic_2E_3C_3D \\
& \quad \quad V1m) V0n)))))))))
\end{aligned}$$

(103)

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (104)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (105)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) (ap c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& (ap c_2Earithmic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))))) \\
& \hspace{15em} (108)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n))) \Leftrightarrow \\
& True) \wedge (((p (ap (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))))))))))))) \\
& \hspace{15em} (109)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmic_2ENUMERAL (ap (ap (ap c_2Earithmic_2E_2D V0n) \\
& V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n)) (ap c_2Earithmic_2ENUMERAL (ap (ap (ap c_2Eenumeral_2EiSUB \\
& c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0)))) \\
& \hspace{15em} (110)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap c_2Enum_2ESUC V0m) = (ap c_2Enum_2ESUC V1n)) \Leftrightarrow (V0m = V1n))) \\
& \hspace{15em} (111)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \quad (112)$$

Assume the following.

$$((ap c_2Erat_2Erat_of_num c_2Enum_2E0) = (ap c_2Erat_2Eabs_rat c_2Efrac_2Efrac_0)) \quad (113)$$

Assume the following.

$$(\forall V0x \in ty_2Efrac_2Efrac. ((ap c_2Erat_2Eabs_rat (ap c_2Efrac_2Efrac_ainv (ap c_2Erat_2Erep_rat (ap c_2Erat_2Eabs_rat V0x)))) = (ap c_2Erat_2Eabs_rat (ap c_2Efrac_2Efrac_ainv V0x)))) \quad (114)$$

Assume the following.

$$(\forall V0f1 \in ty_2Efrac_2Efrac. (\forall V1f2 \in ty_2Efrac_2Efrac. ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Eabs_rat V0f1) (ap c_2Erat_2Eabs_rat V1f2))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_lt (ap (ap c_2Einteger_2Eint_mul (ap c_2Efrac_2Efrac_nmr V0f1) (ap c_2Efrac_2Efrac_dnm V1f2))) (ap (ap c_2Einteger_2Eint_mul (ap c_2Efrac_2Efrac_nmr V1f2) (ap c_2Efrac_2Efrac_dnm V0f1)))))))))) \quad (115)$$

Assume the following.

$$(\forall V0f1 \in ty_2Efrac_2Efrac. (\forall V1f2 \in ty_2Efrac_2Efrac. ((p (ap (ap c_2Erat_2Erat_leq (ap c_2Erat_2Eabs_rat V0f1) (ap c_2Erat_2Eabs_rat V1f2))) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap (ap c_2Einteger_2Eint_mul (ap c_2Efrac_2Efrac_nmr V0f1) (ap c_2Efrac_2Efrac_dnm V1f2))) (ap (ap c_2Einteger_2Eint_mul (ap c_2Efrac_2Efrac_nmr V1f2) (ap c_2Efrac_2Efrac_dnm V0f1)))))))))) \quad (116)$$

Assume the following.

$$(\forall V0n1 \in ty_2Enum_2Enum. ((ap c_2Erat_2Erat_of_num V0n1) = (ap c_2Erat_2Eabs_rat (ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) (ap c_2Einteger_2Eint_of_num V0n1)) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))))) \quad (117)$$

Assume the following.

$$(\forall V0a \in ty_2Enum_2Enum. (\forall V1b \in ty_2Enum_2Enum. (p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num V0a) (ap c_2Erat_2Erat_of_num V1b))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0a) V1b)))))) \quad (118)$$

Assume the following.

$$(\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (ap (ap c_2Erat_2Erat_add V0a) V1b) = (ap (ap c_2Erat_2Erat_add V1b) V0a)))) \quad (119)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. ((ap (ap c_2Erat_2Erat_sub V0r1) V1r2) = (ap (ap c_2Erat_2Erat_add V0r1) (ap c_2Erat_2Erat_ainv V1r2)))))) \quad (120)$$

Assume the following.

$$((ap c_2Erat_2Erat_ainv (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) = (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) \quad (121)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. ((ap c_2Erat_2Erat_ainv (ap (ap c_2Erat_2Erat_add V0r1) V1r2)) = (ap (ap c_2Erat_2Erat_add (ap c_2Erat_2Erat_ainv V0r1)) (ap c_2Erat_2Erat_ainv V1r2)))))) \quad (122)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. (((ap c_2Erat_2Erat_ainv V0r1) = V1r2) \Leftrightarrow (V0r1 = (ap c_2Erat_2Erat_ainv V1r2)))))) \quad (123)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. (((ap c_2Erat_2Erat_ainv V0r1) = (ap c_2Erat_2Erat_ainv V1r2)) \Leftrightarrow (V0r1 = V1r2)))) \quad (124)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. ((p (ap (ap c_2Erat_2Erat_les V0r1) V1r2)) \Rightarrow (\neg (p (ap (ap c_2Erat_2Erat_les V1r2) V0r1)))))) \quad (125)$$

Assume the following.

$$(\forall V0r1 \in ty_2Erat_2Erat. (p (ap (ap c_2Erat_2Erat_leq V0r1) V0r1))) \quad (126)$$

Assume the following.

$$(p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) (ap c_2Erat_2Erat_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \quad (127)$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (\\
& \quad \forall V2c \in ty_2Erat_2Erat. (((p (ap (ap c_2Erat_2Erat_les V0a) \\
& V1b)) \wedge (p (ap (ap c_2Erat_2Erat_leq V1b) V2c))) \Rightarrow (p (ap (ap c_2Erat_2Erat_les \\
& \quad V0a) V2c))))))
\end{aligned} \tag{128}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Erat_2Erat. (\forall V1b \in ty_2Erat_2Erat. (\\
& \quad \forall V2c \in ty_2Erat_2Erat. (((p (ap (ap c_2Erat_2Erat_leq V0a) \\
& V1b)) \wedge (p (ap (ap c_2Erat_2Erat_les V1b) V2c))) \Rightarrow (p (ap (ap c_2Erat_2Erat_les \\
& \quad V0a) V2c))))))
\end{aligned} \tag{129}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num c_2Enum_2E0)) \\
& V0r1)) \Rightarrow ((p (ap (ap c_2Erat_2Erat_leq (ap c_2Erat_2Erat_of_num \\
& c_2Enum_2E0)) V1r2)) \Rightarrow (p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erat_2Erat_add V0r1) V1r2))))))
\end{aligned} \tag{130}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& (\forall V2r3 \in ty_2Erat_2Erat. ((V0r1 = (ap (ap c_2Erat_2Erat_sub \\
& V1r2) V2r3)) \Leftrightarrow ((ap (ap c_2Erat_2Erat_add V0r1) V2r3) = V1r2))))))
\end{aligned} \tag{131}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& (\forall V2r3 \in ty_2Erat_2Erat. (((ap (ap c_2Erat_2Erat_add V0r1) \\
& V2r3) = (ap (ap c_2Erat_2Erat_add V1r2) V2r3)) \Leftrightarrow (V0r1 = V1r2))))))
\end{aligned} \tag{132}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_ainv V0r1)) \\
& (ap c_2Erat_2Erat_ainv V1r2))) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_les \\
& \quad V1r2) V0r1))))))
\end{aligned} \tag{133}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r1 \in ty_2Erat_2Erat. (\forall V1r2 \in ty_2Erat_2Erat. \\
& ((p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_ainv V0r1)) \\
& V1r2)) \Leftrightarrow (p (ap (ap c_2Erat_2Erat_les (ap c_2Erat_2Erat_ainv \\
& \quad V1r2)) V0r1))))))
\end{aligned} \tag{134}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0n \in A_{27a}. (((ap \text{ c_2Erat_2Erat_of_num} \\ & \quad \text{c_2Enum_2E0}) = \text{c_2Erat_2Erat_0}) \wedge (\forall V1n \in \text{ty_2Enum_2Enum}. \\ & ((ap \text{ c_2Erat_2Erat_of_num } (ap \text{ c_2Enum_2ESUC } V1n)) = (ap (ap \text{ c_2Erat_2Erat_add} \\ & \quad (ap \text{ c_2Erat_2Erat_of_num } V1n)) \text{ c_2Erat_2Erat_1})))))) \end{aligned} \quad (135)$$

Assume the following.

$$(\text{c_2Erat_2Erat_0} = (ap \text{ c_2Erat_2Erat_of_num } \text{c_2Enum_2E0})) \quad (136)$$

Assume the following.

$$(\text{c_2Erat_2Erat_1} = (ap \text{ c_2Erat_2Erat_of_num } (ap \text{ c_2Earithmic_2ENUMERAL} \\ (ap \text{ c_2Earithmic_2EBIT1 } \text{c_2Earithmic_2EZERO})))) \quad (137)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p \text{ V0t}))) \Leftrightarrow (p \text{ V0t}))) \quad (138)$$

Assume the following.

$$(\forall V0A \in 2. ((p \text{ V0A}) \Rightarrow ((\neg(p \text{ V0A})) \Rightarrow \text{False}))) \quad (139)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \text{ V0A}) \vee (p \text{ V1B}))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p \text{ V0A}) \Rightarrow \text{False}) \Rightarrow ((\neg(p \text{ V1B})) \Rightarrow \text{False})))) \quad (140)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p \text{ V0A})) \vee (p \text{ V1B}))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p \text{ V0A}) \Rightarrow ((\neg(p \text{ V1B})) \Rightarrow \text{False})))))) \quad (141)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p \text{ V0A})) \Rightarrow \text{False}) \Rightarrow (((p \text{ V0A}) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (142)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \text{ V0p}) \Leftrightarrow (\\ & (p \text{ V1q}) \Leftrightarrow (p \text{ V2r}))) \Leftrightarrow (((p \text{ V0p}) \vee ((p \text{ V1q}) \vee (p \text{ V2r}))) \wedge (((p \text{ V0p}) \vee ((\neg \\ & p \text{ V2r})) \vee (\neg(p \text{ V1q})))) \wedge (((p \text{ V1q}) \vee ((\neg(p \text{ V2r})) \vee (\neg(p \text{ V0p})))) \wedge ((p \text{ V2r}) \vee \\ & ((\neg(p \text{ V1q})) \vee (\neg(p \text{ V0p})))))))))) \end{aligned} \quad (143)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \text{ V0p}) \Leftrightarrow (\\ & (p \text{ V1q}) \wedge (p \text{ V2r}))) \Leftrightarrow (((p \text{ V0p}) \vee ((\neg(p \text{ V1q})) \vee (\neg(p \text{ V2r})))) \wedge (((p \text{ V1q}) \vee \\ & (\neg(p \text{ V0p})) \wedge ((p \text{ V2r}) \vee (\neg(p \text{ V0p})))))))))) \end{aligned} \quad (144)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{145}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{146}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{147}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{148}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{149}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V0p))) \tag{150}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{151}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{152}$$

Theorem 1

$$\begin{aligned}
& (\forall V0i \in ty_2Einteger_2Eint. (\forall V1j \in ty_2Einteger_2Eint. \\
& ((p \ (ap \ (ap \ c_2Erat_2Erat_les \ (ap \ c_2Erat_2Erat_of_int \ V0i)) \\
& (ap \ c_2Erat_2Erat_of_int \ V1j))) \Leftrightarrow (p \ (ap \ (ap \ c_2Einteger_2Eint_lt \\
& \ V0i) \ V1j))))))
\end{aligned}$$