thm_2EreaderMonad_2EBIND__JOIN (TM-cHT73fPPR66ov4TGGGf8ZbTU3GyjYLdmi)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2EreaderMonad_2EBIND *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27s : \iota.\lambda V0M \in (A\_27a^{A}$

Definition 5 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 6 *We define* c_2EreaderMonad_2EFMAP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27s : \iota.\lambda V0f \in (A\_27b^{A}$

Definition 7 *We define* c_2EreaderMonad_2EJOIN *to be* $\lambda A\_27a : \iota.\lambda A\_27s : \iota.\lambda V0MM \in ((A\_27a^{A\_27s})^{A\_2}$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}).(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{5}$$

Theorem 1

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0M \in (A\_27c^{A\_27a}).(\forall V1f \in ((A\_27b^{A\_27a})^{A\_27c}).\ ((ap\ (ap\ (\mathsf{c\_2EreaderMonad\_2EBIND}\ A\_27c\ A\_27b\ A\_27a)\ V0M)\ V1f) = (ap\ (\mathsf{c\_2EreaderMonad\_2EJOIN}\ A\_27b\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2EreaderMonad\_2EFMAP}\ A\_27c\ (A\_27b^{A\_27a})\ A\_27a)\ V1f)\ V0M)))))$$