

thm_2EreaderMonad_2EFMAP__BIND (TMbv6jfyAqDbgdxrFKSxk6VFB2X4kPvgQHr)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 4 We define $c_2EreaderMonad_2EBIND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27s : \iota.\lambda V0M \in (A_27a^{A_27b})$

Definition 5 We define $c_2EreaderMonad_2EUNIT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1s \in A_27b$

Definition 6 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 7 We define $c_2EreaderMonad_2EFMAP$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27s : \iota.\lambda V0f \in (A_27b^{A_27a})$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V1x)))) \tag{2}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{3}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \tag{4}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c.nonempty A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}).(\forall V2x \in A_27c.((ap (ap (c_2Ecombin_2Eo A_27c A_27b A_27a) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \tag{5}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27c}). (\forall V1M \in (A_27c^{A_27a}). \\ & ((ap\ (ap\ (c_2EreaderMonad_2EFMAP\ A_27c\ A_27b\ A_27a)\ V0f)\ V1M) = \\ & (ap\ (ap\ (c_2EreaderMonad_2EBIND\ A_27c\ A_27b\ A_27a)\ V1M)\ (ap\ (ap \\ & (c_2Ecombin_2Eo\ A_27c\ (A_27b^{A_27a})\ A_27b)\ (c_2EreaderMonad_2EUNIT \\ & A_27b\ A_27a))\ V0f)))))) \end{aligned}$$