

thm_2Ereal_2EPOW__1 (TMGt2wJq63TgGesdTLGagEJpmjqdhtf1e1X)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)\ ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \tag{5}$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \quad (8)$$

Let $c_2Eenum_2EERP_num : \iota$ be given. Assume the following.

$$c_2Eenum_2EERP_num \in (\omega^{ty_2Eenum_2Eenum}) \quad (9)$$

Let $c_2Eenum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Eenum_2ESUC_REP \in (\omega^{\omega}) \quad (10)$$

Let $c_2Eenum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Eenum_2EABS_num \in (ty_2Eenum_2Eenum)^{\omega} \quad (11)$$

Definition 8 We define c_2Eenum_2ESUC to be $\lambda V0m \in ty_2Eenum_2Eenum.(ap\ c_2Eenum_2EABS_num)$

Let $c_2Eenum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Eenum_2EZERO_REP \in \omega \quad (12)$$

Definition 9 We define c_2Eenum_2E0 to be $(ap\ c_2Eenum_2EABS_num\ c_2Eenum_2EZERO_REP)$.

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Eenum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Eenum_2Eenum)^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum} \quad (13)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Eenum_2Eenum.(ap\ (ap\ c_2Earithmetic$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Eenum_2Eenum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Eenum_2Eenum} \quad (14)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Eenum_2Eenum})^{ty_2Erealax_2Ereal} \quad (15)$$

Definition 13 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 14 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$\begin{aligned} ((ap\ c_2Earithmetic_2ENUMERAL (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)) = \\ (ap\ c_2Enum_2ESUC\ c_2Enum_2E0)) \end{aligned} \tag{16}$$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{18}$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Erealax_2Ereal.((ap (ap\ c_2Erealax_2Ereal_mul \\ V0x) (ap\ c_2Ereal_2Ereal_of_num (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) = V0x)) \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned} ((\forall V0x \in ty_2Erealax_2Ereal.((ap (ap\ c_2Ereal_2Epow\ V0x) \\ c_2Enum_2E0) = (ap\ c_2Ereal_2Ereal_of_num (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \wedge (\forall V1x \in \\ ty_2Erealax_2Ereal.(\forall V2n \in ty_2Enum_2Enum.((ap (ap\ c_2Ereal_2Epow \\ V1x) (ap\ c_2Enum_2ESUC\ V2n)) = (ap (ap\ c_2Erealax_2Ereal_mul\ V1x) \\ (ap (ap\ c_2Ereal_2Epow\ V1x)\ V2n)))))) \end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned} (\forall V0x \in ty_2Erealax_2Ereal.((ap (ap\ c_2Ereal_2Epow\ V0x) \\ (ap\ c_2Earithmetic_2ENUMERAL (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = \\ V0x)) \end{aligned}$$