

thm_2Ereal_2EPOW__2__LT (TMSX- ouachw4grZbBKwEMMdBQpnWWA16uhYX)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum})ty_2Eenum_2Eenum) \tag{2}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{3}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{4}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{5}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})ty_2Erealax_2Ereal) \tag{6}$$

Definition 7 We define $c_Emin_E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal.(ap (c_Emin_E_40 (ty$
Let $c_Erealax_Etrealm_add : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_add \in (((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)} \quad (7)$$

Let $c_Erealax_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_eq \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)} \quad (8)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)}} \quad (9)$$

Definition 9 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod ty_Ehreal_Ehreal ty$

Definition 10 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax$

Let $c_Erealax_Etrealm_lt : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_lt \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)} \quad (10)$$

Definition 11 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax$

Let $c_Erealax_Etrealm_mul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_mul \in (((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})_{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)} \quad (11)$$

Definition 12 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax$

Let $c_Eenum_EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_EREP_num \in (\omega^{ty_Eenum_Eenum}) \quad (12)$$

Let $c_Eenum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Let $c_Eenum_EABS_num : \iota$ be given. Assume the following.

$$c_Eenum_EABS_num \in (ty_Eenum_Eenum^{\omega}) \quad (14)$$

Definition 13 We define c_Eenum_ESUC to be $\lambda V0m \in ty_Eenum_Eenum.(ap c_Eenum_EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{15}$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.))$

Definition 16 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \tag{16}$$

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 18 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)$

Definition 19 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{17}$$

Definition 20 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ V0m) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \tag{18}$$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \tag{20}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& (ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num \\
& (ap (ap c_2Earithmic_2E_2B V0m) V1n))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add \\
& V0x) V0x) = (ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO)))) \\
& V0x)))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0w \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal. \\
& (\forall V2y \in ty_2Erealax_2Ereal.(\forall V3z \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Erealax_2Ereal_lt V0w) V1x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& V2y) V3z))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_add \\
& V0w) V2y)) (ap (ap c_2Erealax_2Ereal_add V1x) V3z))))))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Ereal_2Epow V0x) \\
& c_2Enum_2E0) = (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \wedge (\forall V1x \in \\
& ty_2Erealax_2Ereal.(\forall V2n \in ty_2Enum_2Enum.((ap (ap c_2Ereal_2Epow \\
& V1x) (ap c_2Enum_2ESUC V2n)) = (ap (ap c_2Erealax_2Ereal_mul V1x) \\
& (ap (ap c_2Ereal_2Epow V1x) V2n))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) (ap (ap c_2Ereal_2Epow \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) V0n)))) \\
& \hspace{15em} (29)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num V0n)) (ap (ap c_2Ereal_2Epow (ap \\
& c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO)))) V0n))))
\end{aligned}$$