

# thm\_2Ereal\_2EPOW\_\_NZ (TMGLYKM- BysKzyUBuJM61JDV7JrmCD6gJvFT)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_5C\_2E\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax\_2Ereal) \tag{4}$$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 9** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 (ty\_2Erealax\_2Ereal\_mul) (c\_2Erealax\_2Ereal\_mul)))$ .  
Let  $c\_2Erealax\_2Ereal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal) \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))} \quad (7)$$

**Definition 10** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$ .

**Definition 11** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (8)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (11)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (c\_2Enum\_2ESUC\_REP))$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (12)$$

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 14** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (13)$$

**Definition 15** We define `c2Earithmic2EBIT1` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic2E0) V0n)$

**Definition 16** We define `c2Earithmic2ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let `c2Ereal2Ereal_of_num` :  $\iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (14)$$

Let `c2Ereal2Epow` :  $\iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (15)$$

**Definition 17** We define `c2Ebool2E2F5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool2E21) 2) (\lambda V2t \in 2.(ap (c\_2Ebool2E21) 2) V2t) V1t2) V0t1))$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \quad (21)$$

Assume the following.

$$(\neg((ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)))) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \quad (22)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) \Leftrightarrow ((V0x = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \vee \\
& (V1y = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Epow V0x) \\
& c\_2Enum\_2E0) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge (\forall V1x \in \\
& ty\_2Erealax\_2Ereal. (\forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ereal\_2Epow \\
& V1x) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Erealax\_2Ereal\_mul V1x) \\
& (ap (ap c\_2Ereal\_2Epow V1x) V2n))))))
\end{aligned} \tag{24}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0c \in ty\_2Erealax\_2Ereal. (\forall V1n \in ty\_2Enum\_2Enum. \\
& ((\neg (V0c = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \Rightarrow (\neg ((ap \\
& (ap c\_2Ereal\_2Epow V0c) V1n) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned}$$