

thm_2Ereal_2EREAL__ADD__LID__UNIQ (TMUF5A53YN4NeKvfbshorStc4i64whZunEd)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o(x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{4}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{5}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{6}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{7}$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \tag{8}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealx_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (9)$$

Let $c_2Erealx_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (10)$$

Let $c_2Erealx_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_ABS_CLASS \in (ty_2Erealx_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (11)$$

Definition 7 We define $c_2Erealx_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealx_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal.((ap (ap c_2Erealx_2Ereal_add (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x)) \quad (13)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal.(\forall V1y \in ty_2Erealx_2Ereal.(\forall V2z \in ty_2Erealx_2Ereal.(((ap (ap c_2Erealx_2Ereal_add (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V1y) V2z) \Leftrightarrow (V0x = V1y)))))) \quad (14)$$

Theorem 1

$$(\forall V0x \in ty_2Erealx_2Ereal.(\forall V1y \in ty_2Erealx_2Ereal.(((ap (ap c_2Erealx_2Ereal_add V0x) V1y) = V1y) \Leftrightarrow (V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))$$