

thm_2Ereal_2EREAL__DIV__DENOM__CANCEL
(TMR-
WvwkgBE3wgiLSsHwDhsRB9YYeCaaf4H8)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (6)$$

Let $c_2Erealax_2Ereal_2EABS_2ECLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_2EABS_2ECLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_2EABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_2EABS)$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (8)$$

Definition 8 We define $c_2Erealax_2Ereal_2Emul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 9 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(a$

Let $c_2Enum_2EZERO_2REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_2REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_2enum : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_2enum \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_2enum\ c_2Enum_2EZERO_2REP)$.

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_2num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_2num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_2REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_2REP \in (\omega^{\omega}) \quad (13)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_2enum$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 13 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic$

Definition 14 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (15)$$

Definition 15 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 16 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg (p V0t)))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg (\neg (p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & ((ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y) = (ap\ (ap\ c_2Erealax_2Ereal_mul \\ & V1y)\ V0x)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2z \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul \\ & V0x)\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1y)\ V2z)) = (ap\ (ap\ c_2Erealax_2Ereal_mul \\ & (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y))\ V2z)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\neg(V0x = (ap\ c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0))) \Rightarrow ((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Erealax_2Einv \\ & V0x))\ V0x) = (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmic_2ENUMERAL \\ & (ap\ c_2Earithmic_2EBIT1\ c_2Earithmic_2EZERO)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & ((ap\ c.\mathit{Erealax_2Einv}\ (ap\ c.\mathit{Ereal_2Ereal_of_num}\ c.\mathit{Eenum_2E0})) = \\ & \quad (ap\ c.\mathit{Ereal_2Ereal_of_num}\ c.\mathit{Eenum_2E0})) \end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.\mathit{2Erealax_2Ereal}.\ ((ap\ (ap\ c.\mathit{Erealax_2Ereal_mul}\ \\ & V0x)\ (ap\ c.\mathit{Ereal_2Ereal_of_num}\ (ap\ c.\mathit{Earithmic_2ENUMERAL}\ \\ & (ap\ c.\mathit{Earithmic_2EBIT1}\ c.\mathit{Earithmic_2EZERO})))) = V0x)) \end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.\mathit{2Erealax_2Ereal}.\ ((ap\ (ap\ c.\mathit{Erealax_2Ereal_mul}\ \\ & (ap\ c.\mathit{Ereal_2Ereal_of_num}\ c.\mathit{Eenum_2E0}))\ V0x) = (ap\ c.\mathit{Ereal_2Ereal_of_num}\ \\ & \quad c.\mathit{Eenum_2E0}))) \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.\mathit{2Erealax_2Ereal}.\ ((ap\ (ap\ c.\mathit{Erealax_2Ereal_mul}\ \\ & V0x)\ (ap\ c.\mathit{Ereal_2Ereal_of_num}\ c.\mathit{Eenum_2E0})) = (ap\ c.\mathit{Ereal_2Ereal_of_num}\ \\ & \quad c.\mathit{Eenum_2E0}))) \end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.\mathit{2Erealax_2Ereal}.\ ((\neg(V0x = (ap\ c.\mathit{Ereal_2Ereal_of_num}\ \\ & \quad c.\mathit{Eenum_2E0}))) \Rightarrow ((ap\ c.\mathit{Erealax_2Einv}\ (ap\ c.\mathit{Erealax_2Einv}\ V0x)) = \\ & \quad V0x))) \end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.\mathit{2Erealax_2Ereal}.\ (((ap\ c.\mathit{Erealax_2Einv}\ V0x) = \\ & (ap\ c.\mathit{Ereal_2Ereal_of_num}\ c.\mathit{Eenum_2E0})) \Leftrightarrow (V0x = (ap\ c.\mathit{Ereal_2Ereal_of_num}\ \\ & \quad c.\mathit{Eenum_2E0})))) \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.\mathit{2Erealax_2Ereal}.\ (\forall V1y \in ty.\mathit{2Erealax_2Ereal}.\ \\ & (((\neg(V0x = (ap\ c.\mathit{Ereal_2Ereal_of_num}\ c.\mathit{Eenum_2E0}))) \wedge (\neg(V1y = \\ & (ap\ c.\mathit{Ereal_2Ereal_of_num}\ c.\mathit{Eenum_2E0})))) \Rightarrow ((ap\ c.\mathit{Erealax_2Einv}\ \\ & (ap\ (ap\ c.\mathit{Erealax_2Ereal_mul}\ V0x)\ V1y)) = (ap\ (ap\ c.\mathit{Erealax_2Ereal_mul}\ \\ & \quad (ap\ c.\mathit{Erealax_2Einv}\ V0x))\ (ap\ c.\mathit{Erealax_2Einv}\ V1y)))))) \end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.\ ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.\ ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow \mathit{False}))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q) \vee (p V2r)) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(((\neg(\neg(p V0p))) \Rightarrow (p V0p)))) \quad (50)$$

Theorem 1

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.((\neg(V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \Rightarrow ((ap (ap c_2Ereal_2E_2F (ap (ap c_2Ereal_2E_2F V1y) V0x)) (ap (ap c_2Ereal_2E_2F V2z) V0x)) = (ap (ap c_2Ereal_2E_2F V1y) V2z)))))))$$