

# thm\_2Ereal\_2EREAL\_\_ENTIRE (TMRsMzPFcBPUY5c2ZgAQoFLpCdT41PpnuBE)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 10** We define  $c\_Enum\_ESUC$  to be  $\lambda V0m \in ty\_Enum\_Enum.(ap\ c\_Enum\_EABS\_num$   
Let  $c\_Earithmic\_E\_B : \iota$  be given. Assume the following.

$$c\_Earithmic\_E\_B \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (6)$$

**Definition 11** We define  $c\_Earithmic\_EBIT1$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap\ (ap\ c\_Earithmic$

**Definition 12** We define  $c\_Earithmic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Enum\_Enum.V0x$ .

Let  $ty\_Ehreal\_Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_Ehreal\_Ehreal \quad (7)$$

Let  $ty\_Epair\_Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_Epair\_Eprod\ A0\ A1) \quad (8)$$

Let  $ty\_Erealax\_Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_Erealax\_Ereal \quad (9)$$

Let  $c\_Erealax\_Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_REP\_CLASS \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{ty\_Erealax\_Ereal}) \quad (10)$$

**Definition 13** We define  $c\_Emin\_E\_40$  to be  $\lambda A.\lambda P \in 2^A$ .**if**  $(\exists x \in A.p\ (ap\ P\ x))$  **then**  $(the\ (\lambda x.x \in A \wedge P\ x))$   
of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_Erealax\_Ereal\_REP$  to be  $\lambda V0a \in ty\_Erealax\_Ereal.(ap\ (c\_Emin\_E\_40\ (the\ (\lambda x.x \in A \wedge P\ x))))$

Let  $c\_Erealax\_Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_inv \in ((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal}) \quad (11)$$

Let  $c\_Erealax\_Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_eq \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal}) \quad (12)$$

Let  $c\_Erealax\_Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_ABS\_CLASS \in (ty\_Erealax\_Ereal)^{(2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})} \quad (13)$$

**Definition 15** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)$

**Definition 16** We define  $c\_Erealax\_Einv$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal\_ABS\ (the\ (\lambda x.x \in A \wedge P\ x))))$

**Definition 17** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (14)$$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} \quad (15)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul V0x) (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) V2z)))))) \quad (23)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((\neg(V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \Rightarrow ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Einv \\
& V0x)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul \\
& V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{27}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) \Leftrightarrow ((V0x = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \vee \\
& (V1y = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))
\end{aligned}$$