

thm_2Ereal_2EREAL_INF_LT
(TMZ2iykv7s8ZWFhQG R1RUQeDau5vD4AACBY)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_K$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 4 We define $c_2Ecombin_2E_S$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2E_I$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2E_S A_27a (A_27a^{A_27a})) A_27a))$

Let $ty_2Ehreal_2E_hreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2E_hreal \tag{1}$$

Let $ty_2Epair_2E_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2E_prod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2E_real : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2E_real \tag{3}$$

Let $c_2Erealax_2E_real_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2E_real_REP_CLASS \in ((2^{(ty_2Epair_2E_prod\ ty_2Ehreal_2E_hreal\ ty_2Ehreal_2E_hreal)})\ ty_2Erealax_2E_real) \tag{4}$$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap\ P\ x)) \mathbf{then} (the (\lambda x. x \in A \wedge p (ap\ P\ x)))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)) P))$

Definition 8 We define $c_2Erealax_2E_real_REP$ to be $\lambda V0a \in ty_2Erealax_2E_real. (ap (c_2Emin_2E_40 (ty_2Erealax_2E_real\ V0a)))$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (5)$$

Definition 9 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (6)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (8)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 11 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 15 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Emin_2E_40\ ty_2Ereal$

Definition 16 We define c_2Ereal_2Einf to be $\lambda V0p \in (2^{ty_2Erealax_2Ereal}).(ap\ c_2Erealax_2Ereal_neg\ (ap\ c$

Definition 17 We define $c_2Ebool_2E_F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 18 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 19 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((p\ V0P) \vee (\exists V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x))))))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q)))))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (25)$$

Assume the following.

$$(\forall V0p \in (2^{ty_2Erealax_2Ereal}). (\forall V1r \in ty_2Erealax_2Ereal. (((\exists V2x \in ty_2Erealax_2Ereal. (p\ (ap\ V0p\ V2x))) \wedge (\forall V3x \in ty_2Erealax_2Ereal. ((p\ (ap\ V0p\ V3x)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V1r)\ V3x)))))) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V1r)\ (ap\ c_2Ereal_2Einf\ V0p)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B)) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (36)$$

Theorem 1

$$\begin{aligned} & (\forall V0p \in (2^{ty_2Erealax_2Ereal}).(\forall V1z \in ty_2Erealax_2Ereal. \\ & (((\exists V2x \in ty_2Erealax_2Ereal.(p (ap V0p V2x))) \wedge (p (ap (ap \\ & c_2Erealax_2Ereal_lt (ap c_2Ereal_2Einf V0p)) V1z))) \Rightarrow (\exists V3x \in \\ & ty_2Erealax_2Ereal.((p (ap V0p V3x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt \\ & V3x) V1z)))))) \end{aligned}$$