

thm_2Ereal_2EREAL__INV__0 (TMHvrhoaM3oRuyaJzs5QHghAPSzsojRM46X)

October 26, 2020

Let $c_2Enum_2ZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2ZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{4}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{5}$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow p\ Q)$ of type ι .

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1t \in 2.V1t))\ (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))\ (\lambda V3t \in 2.V3t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Definition 8 We define $c_2Ehrat_2Etratl_1$ to be $(ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum\ ty_2Enum_2Enum))$

Let $c_2Ehrat_2Etratl_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etratl_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (8)$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehrat_2Ehrat \quad (9)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (10)$$

Definition 9 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 10 We define $c_2Ehrat_2Ehrat_1$ to be $(ap\ c_2Ehrat_2Ehrat_ABS\ c_2Ehrat_2Etratl_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Ehrat_2Ehrat}) \quad (11)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 12 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat. (ap\ (c_2Emin_2E_40\ (ty_2$

Let $c_2Ehrat_2Etratl_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etratl_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (12)$$

Definition 13 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat. \lambda V1T2 \in ty_2Ehrat_2Ehrat$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ (ty_2$

Definition 15 We define $c_Ehreal_Ehreal_lt$ to be $\lambda V0x \in ty_Ehreal_Ehreal. \lambda V1y \in ty_Ehreal_Ehreal$

Definition 16 We define $c_Ehreal_Ecut_of_hrat$ to be $\lambda V0x \in ty_Ehreal_Ehreal. (\lambda V1y \in ty_Ehreal_Ehreal$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \quad (13)$$

Let $c_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$c_Ehreal_Ehreal \in (ty_Ehreal_Ehreal^{(2^{ty_Ehreal_Ehreal})}) \quad (14)$$

Definition 17 We define $c_Ehreal_Ehreal_1$ to be $(ap\ c_Ehreal_Ehreal\ (ap\ c_Ehreal_Ecut_of_hrat\ ty_Ehreal_Ehreal))$

Definition 18 We define $c_Erealax_Etreax_0$ to be $(ap\ (ap\ (c_Epair_E2C\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal))\ ty_Ehreal_Ehreal)$

Let $c_Erealax_Etreax_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etreax_eq \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal)}) \quad (15)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal^{(2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})}) \quad (16)$$

Definition 19 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)$

Definition 20 We define $c_Erealax_Ereal_0$ to be $(ap\ c_Erealax_Ereal_ABS\ c_Erealax_Etreax_0)$.

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealax_Ereal}) \quad (17)$$

Definition 21 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal. (ap\ (c_Emin_E40\ ty_Ehreal_Ehreal))$

Let $c_Erealax_Ereal_inv : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_inv \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal}) \quad (18)$$

Definition 22 We define $c_Erealax_Ereal_inv$ to be $\lambda V0T1 \in ty_Erealax_Ereal. (ap\ c_Erealax_Ereal_ABS\ ty_Ehreal_Ehreal)$

Assume the following.

$$(c_Erealax_Ereal_0 = (ap\ c_Ereal_Ereal_of_num\ c_Eenum_E0)) \quad (19)$$

Assume the following.

$$((ap\ c_Erealax_Ereal_inv\ c_Erealax_Ereal_0) = c_Erealax_Ereal_0) \quad (20)$$

Theorem 1

$$((ap\ c_Erealax_Ereal_inv\ (ap\ c_Ereal_Ereal_of_num\ c_Eenum_E0)) = (ap\ c_Ereal_Ereal_of_num\ c_Eenum_E0))$$