

thm_2Ereal_2EREAL__LE__DOUBLE
(TMKK6sGKnuJ2G41gk4ougmpidvVSbKbW6BT)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $c_2Enum_2E_2ZERO_2E_2REP : \iota$ be given. Assume the following.

$$c_2Enum_2E_2ZERO_2E_2REP \in \omega \tag{1}$$

Let $ty_2Enum_2E_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2E_2Enum \tag{2}$$

Let $c_2Enum_2E_2ABS_2E_2num : \iota$ be given. Assume the following.

$$c_2Enum_2E_2ABS_2E_2num \in (ty_2Enum_2E_2Enum^{\omega}) \tag{3}$$

Definition 5 We define $c_2Enum_2E_2E0$ to be $(ap\ c_2Enum_2E_2ABS_2E_2num\ c_2Enum_2E_2ZERO_2E_2REP)$.

Let $ty_2Erealx_2E_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2E_2Ereal \tag{4}$$

Let $c_2Ereal_2E_2Ereal_2E_2of_2E_2num : \iota$ be given. Assume the following.

$$c_2Ereal_2E_2Ereal_2E_2of_2E_2num \in (ty_2Erealx_2E_2Ereal^{ty_2Enum_2E_2Enum}) \tag{5}$$

Definition 6 We define $c_2Emin_2E_3D_2E_3D_2E_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_2E_3D_2E_3E\ V0t)\ c_2Ebool_2E_2E7E))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (7)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (8)$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (9)$$

Definition 10 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (11)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (12)$$

Definition 11 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 12 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 13 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x)) \quad (16)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((\neg (p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt V1y) V0x)))))) \quad (17)$$

Assume the following.

$$(\forall V0w \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal.(\forall V2y \in ty_2Erealax_2Ereal.(\forall V3z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V0w) V1x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V2y) V3z)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_add V0w) V2y)) (ap (ap c_2Erealax_2Ereal_add V1x) V3z)))))))))) \quad (18)$$

Assume the following.

$$(\forall V0w \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal.(\forall V2y \in ty_2Erealax_2Ereal.(\forall V3z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0w) V1x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V2y) V3z)))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add V0w) V2y)) (ap (ap c_2Erealax_2Ereal_add V1x) V3z)))))))))) \quad (19)$$

Theorem 1

$$(\forall V0x \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V0x))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x))))))$$