

thm_2Ereal_2EREAL__LE__RMUL__IMP
(TMaYrUb8CsVNJR6aQdmn61etQEiqv5Ar2sj)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax}) \tag{4}$$

Definition 2 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_27$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ty_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Ereal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal})_{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal})_{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal} \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (10)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (11)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (12)$$

Definition 9 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 10 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21))$

Definition 13 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t)))$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V1y) V0x))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V1y) V2z))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)))))))
\end{aligned} \tag{14}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V1y) V2z))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V1y) V0x)) (ap (ap c_2Erealax_2Ereal_mul V2z) V0x)))))))
\end{aligned}$$