

thm\_2Ereal\_2EREAL\_\_NEG\_\_EQ0  
(TMcRXNE67t74yzjuJoTejbBU5UegVEtaNxd)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{7}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \tag{8}$$

**Definition 4** We define  $c\_2Emin\_2E.40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then** *(the*  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ .

**Definition 5** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E.3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Ebool\_2E.21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E.3D (2^{A-27a}))$

**Definition 7** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal.(ap (c\_2Emin\_2E.40 (ty$

Let  $c\_2Erealx\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} \quad (9)$$

Let  $c\_2Erealx\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} \quad (10)$$

Let  $c\_2Erealx\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_ABS\_CLASS \in (ty\_2Erealx\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}} \quad (11)$$

**Definition 8** We define  $c\_2Erealx\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty$

**Definition 9** We define  $c\_2Erealx\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.(ap c\_2Erealx\_2Ereal$

Let  $c\_2Erealx\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} \quad (12)$$

**Definition 10** We define  $c\_2Erealx\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.\lambda V1T2 \in ty\_2Erealx$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealx\_2Ereal.((ap (ap c\_2Erealx\_2Ereal\_add (ap c\_2Erealx\_2Ereal\_neg V0x)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (14)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealx\_2Ereal.((ap (ap c\_2Erealx\_2Ereal\_add (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = V0x)) \quad (15)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
 & V0x) (ap c\_2Erealax\_2Ereal\_neg V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
 & \quad c\_2Enum\_2E0)))
 \end{aligned}
 \tag{16}$$

**Theorem 1**

$$\begin{aligned}
 & (\forall V0x \in ty\_2Erealax\_2Ereal. (((ap c\_2Erealax\_2Ereal\_neg \\
 & V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \Leftrightarrow (V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
 & \quad c\_2Enum\_2E0))))
 \end{aligned}$$