

thm\_2Ereal\_2EREAL\_\_OF\_\_NUM\_\_POW  
(TMV4LDxj9NtA5PmFhsxBW1M9niSo5oirwve)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{3}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{4}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{5}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{6}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \tag{7}$$

**Definition 4** We define  $c\_2Emin\_2E.40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then**  $(the (\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ ).

**Definition 5** We define  $c\_2Ebool\_2E.21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E.3D (2^{A-27a}$

**Definition 6** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E.40 (ty$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (8)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (10)$$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty$

**Definition 8** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (12)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (13)$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (14)$$

**Definition 10** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 11** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E.2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E.2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (15)$$

**Definition 12** We define  $c\_2\text{Earithmetic\_2EBIT1}$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 13** We define  $c\_2\text{Earithmetic\_2ENUMERAL}$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2\text{Ereal\_2Ereal\_of\_num} : \iota$  be given. Assume the following.

$$c\_2\text{Ereal\_2Ereal\_of\_num} \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2\text{Ereal\_2Epow} : \iota$  be given. Assume the following.

$$c\_2\text{Ereal\_2Epow} \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (17)$$

**Definition 14** We define  $c\_2\text{Ebool\_2E\_2F\_5C}$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\ & V0m) c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO)))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in \\ & ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V1m) (ap c\_2Enum\_2ESUC \\ & V2n)) = (ap (ap c\_2Earithmetic\_2E\_2A V1m) (ap (ap c\_2Earithmetic\_2EEXP \\ & V1m) V2n)))))) \end{aligned} \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ & (ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num \\ & V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n)) = (ap c\_2Ereal\_2Ereal\_of\_num \\ & (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Ereal\_2Epow V0x) \\
c\_2Enum\_2E0) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge (\forall V1x \in \\
ty\_2Erealax\_2Ereal.(\forall V2n \in ty\_2Enum\_2Enum.((ap (ap c\_2Ereal\_2Epow \\
& V1x) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Erealax\_2Ereal\_mul V1x) \\
& (ap (ap c\_2Ereal\_2Epow V1x) V2n))))))
\end{aligned} \tag{24}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\
& (ap (ap c\_2Ereal\_2Epow (ap c\_2Ereal\_2Ereal\_of\_num V0x)) V1n) = \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap (ap c\_2Earithmetic\_2EEXP V0x) \\
& V1n))))))
\end{aligned}$$