

thm_2Ereal_2EREAL__POW__POW
(TMKMPH1KHifGLPEXyaiAiLEpxfsUMb85c9q)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{5}$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Definition 6 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$.

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (8)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \quad (9)$$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2))\ t1)$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (11)$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \quad (12)$$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 12 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap\ (c_2Emin_2E_40\ a))$.

Let $c_2Erealx_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal}) \quad (13)$$

Let $c_2Erealx_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal}) \quad (14)$$

Let $c_2Erealx_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_ABS_CLASS \in (ty_2Erealx_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (15)$$

Definition 13 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Erealax_2Ereal)$

Definition 14 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})^{ty_2Erealax_2Ereal}) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Eenum_2Eenum. (\forall V1n \in ty_2Eenum_2Eenum. (\\ & ((ap (ap c_2Earithmetic_2E_2A\ c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A\ V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A\ (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A\ V0m) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = V0m) \wedge (\\ & ((ap (ap c_2Earithmetic_2E_2A\ (ap c_2Enum_2ESUC\ V0m)) V1n) = (ap \\ & (ap c_2Earithmetic_2E_2B\ (ap (ap c_2Earithmetic_2E_2A\ V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A\ V0m) (ap c_2Enum_2ESUC\ V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B\ V0m) (ap (ap c_2Earithmetic_2E_2A \\ & V0m) V1n)))))))))) \end{aligned} \quad (17)$$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Eenum_2Eenum}). (((p (ap V0P\ c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Eenum_2Eenum. ((p (ap V0P\ V1n)) \Rightarrow (p (ap V0P\ (ap c_2Enum_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Eenum_2Eenum. (p (ap V0P\ V2n)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & ((\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Epow\ V0x) \\ & c_2Enum_2E0) = (ap c_2Ereal_2Ereal_of_num\ (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \wedge (\forall V1x \in \\ & ty_2Erealax_2Ereal. (\forall V2n \in ty_2Eenum_2Eenum. ((ap (ap c_2Ereal_2Epow \\ & V1x) (ap c_2Enum_2ESUC\ V2n)) = (ap (ap c_2Erealax_2Ereal_mul\ V1x) \\ & (ap (ap c_2Ereal_2Epow\ V1x) V2n)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. (\forall V1m \in ty_2Enum_2Enum. \\
& (\forall V2n \in ty_2Enum_2Enum. ((ap (ap c_2Ereal_2Epow V0c) (ap \\
& (ap c_2Earithmetic_2E_2B V1m) V2n)) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Ereal_2Epow V0c) V1m)) (ap (ap c_2Ereal_2Epow V0c) V2n))))))
\end{aligned} \tag{22}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1m \in ty_2Enum_2Enum. \\
& (\forall V2n \in ty_2Enum_2Enum. ((ap (ap c_2Ereal_2Epow (ap (ap c_2Ereal_2Epow \\
& V0x) V1m)) V2n) = (ap (ap c_2Ereal_2Epow V0x) (ap (ap c_2Earithmetic_2E_2A \\
& V1m) V2n))))))
\end{aligned}$$