

thm_2Ereal_2EREAL__SUP__ALLPOS
(TMM5vYFpEpAooWYSkZ5ZeQXGzJq1Bn3dhkp)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{4}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{5}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A))))$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_T$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))))$

Definition 8 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 10 We define $c_2Ehrat_2Etrat_1$ to be $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum$

Let $c_2Ehrat_2Etrat_eq : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_eq \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)) \quad (8)$$

Let $ty_2Ehrat_2Ehrat : \iota$ be given. Assume the following.

$$nonempty ty_2Ehrat_2Ehrat \quad (9)$$

Let $c_2Ehrat_2Ehrat_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_ABS_CLASS \in (ty_2Ehrat_2Ehrat^{(2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}) \quad (10)$$

Definition 11 We define $c_2Ehrat_2Ehrat_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)$

Definition 12 We define $c_2Ehrat_2Ehrat_1$ to be $(ap c_2Ehrat_2Ehrat_ABS c_2Ehrat_2Etrat_1)$.

Let $c_2Ehrat_2Ehrat_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Ehrat_2Ehrat_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})(ty_2Ehrat_2Ehrat_1)) \quad (11)$$

Definition 13 We define $c_2Ehrat_2Ehrat_REP$ to be $\lambda V0a \in ty_2Ehrat_2Ehrat.(ap (c_2Emin_2E_40 (ty_2E$

Let $c_2Ehrat_2Etrat_add : \iota$ be given. Assume the following.

$$c_2Ehrat_2Etrat_add \in (((ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})) \quad (12)$$

Definition 14 We define $c_2Ehrat_2Ehrat_add$ to be $\lambda V0T1 \in ty_2Ehrat_2Ehrat.\lambda V1T2 \in ty_2Ehrat_2E$

Definition 15 We define $c_2Ehreal_2Ehrat_lt$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.\lambda V1y \in ty_2Ehrat_2Ehrat$

Definition 16 We define $c_2Ehreal_2Ecut_of_hrat$ to be $\lambda V0x \in ty_2Ehrat_2Ehrat.(\lambda V1y \in ty_2Ehrat_2E$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (13)$$

Let $c_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$c_2Ehreal_2Ehreal \in (ty_2Ehreal_2Ehreal^{(2^{ty_2Ehreal_2Ehreal})}) \quad (14)$$

Definition 17 We define $c_2Ehreal_2Ehreal_1$ to be $(ap\ c_2Ehreal_2Ehreal\ (ap\ c_2Ehreal_2Ecut_of_hrat\ ty_2Ehreal_2Ehreal))$

Definition 18 We define $c_2Erealax_2Etreax_0$ to be $(ap\ (ap\ (c_2Epair_2E2C\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))$

Let $c_2Erealax_2Etreax_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (16)$$

Definition 19 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 20 We define $c_2Erealax_2Ereal_0$ to be $(ap\ c_2Erealax_2Ereal_ABS\ c_2Erealax_2Etreax_0)$.

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax_2Ereal}) \quad (17)$$

Definition 21 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal))$

Let $c_2Erealax_2Etreax_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 22 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Assume the following.

$$(c_2Erealax_2Ereal_0 = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Erealax_2Ereal}).(((\forall V1x \in ty_2Erealax_2Ereal. \\ & ((p\ (ap\ V0P\ V1x)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ c_2Erealax_2Ereal_0) \\ & V1x)))) \wedge ((\exists V2x \in ty_2Erealax_2Ereal.(p\ (ap\ V0P\ V2x))) \wedge \\ & (\exists V3z \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\ & ((p\ (ap\ V0P\ V4x)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V4x)\ V3z)))))) \Rightarrow \\ & (\exists V5s \in ty_2Erealax_2Ereal.(\forall V6y \in ty_2Erealax_2Ereal. \\ & ((\exists V7x \in ty_2Erealax_2Ereal.((p\ (ap\ V0P\ V7x)) \wedge (p\ (ap\ (ap \\ & c_2Erealax_2Ereal_lt\ V6y)\ V7x)))) \Leftrightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\ & V6y)\ V5s))))))))) \end{aligned} \quad (20)$$

Theorem 1

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Erealax_2Ereal}).((\forall V1x \in ty_2Erealax_2Ereal. \\ & ((p (ap V0P V1x)) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0)) V1x)))) \wedge ((\exists V2x \in ty_2Erealax_2Ereal. (p \\ & (ap V0P V2x))) \wedge (\exists V3z \in ty_2Erealax_2Ereal. (\forall V4x \in \\ & ty_2Erealax_2Ereal. ((p (ap V0P V4x)) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt \\ & V4x) V3z))))))) \Rightarrow (\exists V5s \in ty_2Erealax_2Ereal. (\forall V6y \in \\ & ty_2Erealax_2Ereal. ((\exists V7x \in ty_2Erealax_2Ereal. ((p (\\ & ap V0P V7x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V6y) V7x)))) \Leftrightarrow (p (\\ & ap (ap c_2Erealax_2Ereal_lt V6y) V5s)))))) \end{aligned}$$