

thm_2Ereal_2EREAL__SUP__LE
(TMK5HNjsvHNZkYL3MQnfDd2npgVJQzBTd2i)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (5)$$

Definition 9 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2))$

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ 2))$

Definition 12 We define c_2Ereal_2Esup to be $\lambda V0P \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Emin_2E_40\ ty_2Erealax_2Ereal)$

Definition 13 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & (p\ V0t)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Erealax_2Ereal}).(((\exists V1x \in ty_2Erealax_2Ereal. \\ & (p\ (ap\ V0P\ V1x)))) \wedge (\exists V2z \in ty_2Erealax_2Ereal.(\forall V3x \in \\ & ty_2Erealax_2Ereal.((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\ & V3x)\ V2z))))))) \Rightarrow (\forall V4y \in ty_2Erealax_2Ereal.((\exists V5x \in \\ & ty_2Erealax_2Ereal.((p\ (ap\ V0P\ V5x)) \wedge (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\ & V4y)\ V5x)))))) \Leftrightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V4y)\ (ap\ c_2Ereal_2Esup \\ & V0P)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Erealax_2Ereal}).(((\exists V1x \in ty_2Erealax_2Ereal. \\ & (p\ (ap\ V0P\ V1x)))) \wedge (\exists V2z \in ty_2Erealax_2Ereal.(\forall V3x \in \\ & ty_2Erealax_2Ereal.((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & V3x)\ V2z))))))) \Leftrightarrow ((\exists V4x \in ty_2Erealax_2Ereal.(p\ (ap\ V0P\ V4x))) \wedge \\ & (\exists V5z \in ty_2Erealax_2Ereal.(\forall V6x \in ty_2Erealax_2Ereal. \\ & ((p\ (ap\ V0P\ V6x)) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V6x)\ V5z)))))) \end{aligned} \quad (9)$$

Theorem 1

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Erealax_2Ereal}).(((\exists V1x \in ty_2Erealax_2Ereal. \\ & (p\ (ap\ V0P\ V1x)))) \wedge (\exists V2z \in ty_2Erealax_2Ereal.(\forall V3x \in \\ & ty_2Erealax_2Ereal.((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ & V3x)\ V2z))))))) \Rightarrow (\forall V4y \in ty_2Erealax_2Ereal.((\exists V5x \in \\ & ty_2Erealax_2Ereal.((p\ (ap\ V0P\ V5x)) \wedge (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt \\ & V4y)\ V5x)))))) \Leftrightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V4y)\ (ap\ c_2Ereal_2Esup \\ & V0P)))))) \end{aligned}$$