

thm_2Ereal_2Eadd__rat
(TMThr62CkV9j221jCQmhF9PoB9e62GVHwWG)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Ebool_2ECOND` to be $\lambda A.\lambda a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.(ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 10 We define `c_2Emarker_2Eunint` to be $\lambda A.\lambda a : \iota.\lambda V0x \in A.27a.V0x$.

Let `ty_2Ehreal_2Ehreal` : ι be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let `ty_2Erealax_2Ereal` : ι be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let `c_2Erealax_2Ereal__REP__CLASS` : ι be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$
Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (5)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} \quad (7)$$

Definition 12 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 13 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (8)$$

Definition 14 We define $c_2Erealax_2Ereal_inv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Definition 15 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (9)$$

Definition 16 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (12)$$

Definition 17 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP).$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (13)$$

Definition 18 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E7E$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND\ A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND\ A_27a) V1Q) V3x_27) \\ & V5y_27)))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul\ V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul\ V1y) V0x)))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2z \in ty_2Erealax_2Ereal.(((ap (ap c_2Erealax_2Ereal_add \\ & V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add\ V0x) V2z)) \Leftrightarrow (V1y = V2z)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2z \in ty_2Erealax_2Ereal.(((ap (ap c_2Erealax_2Ereal_add \\ & V0x) V2z) = (ap (ap c_2Erealax_2Ereal_add\ V1y) V2z)) \Leftrightarrow (V0x = V1y)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2z \in ty_2Erealax_2Ereal.(((ap (ap c_2Erealax_2Ereal_add \\ & (ap (ap c_2Ereal_2E_2F\ V1y) V0x)) (ap (ap c_2Ereal_2E_2F\ V2z) V0x)) = \\ & (ap (ap c_2Ereal_2E_2F\ (ap (ap c_2Erealax_2Ereal_add\ V1y) V2z)) \\ & V0x)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_2Erealax_2Ereal.(\forall V1b \in ty_2Erealax_2Ereal. \\ & (\forall V2c \in ty_2Erealax_2Ereal.(\forall V3d \in ty_2Erealax_2Ereal. \\ & (((\neg(V1b = (ap c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \wedge (\neg(V3d = \\ & (ap c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))) \Rightarrow ((ap (ap c_2Erealax_2Ereal_add \\ & (ap (ap c_2Ereal_2E_2F\ V0a) V1b)) (ap (ap c_2Ereal_2E_2F\ V2c) V3d)) = \\ & (ap (ap c_2Ereal_2E_2F\ (ap (ap c_2Erealax_2Ereal_add\ (ap (ap c_2Erealax_2Ereal_mul \\ & V0a) V3d)) (ap (ap c_2Erealax_2Ereal_mul\ V1b) V2c))) (ap (ap c_2Erealax_2Ereal_mul \\ & V1b) V3d)))))))))) \end{aligned} \quad (30)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2u \in ty_2Erealax_2Ereal. (\forall V3v \in ty_2Erealax_2Ereal. \\ & ((ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Ereal_2E_2F V0x) V1y)) \\ & (ap (ap c_2Ereal_2E_2F V2u) V3v)) = (ap (ap (ap (c_2Ebool_2ECOND \\ & ty_2Erealax_2Ereal) (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) \\ V1y) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) (ap (ap c_2Erealax_2Ereal_add \\ & (ap (c_2Emarker_2Eunint ty_2Erealax_2Ereal) (ap (ap c_2Ereal_2E_2F \\ V0x) V1y))) (ap (ap c_2Ereal_2E_2F V2u) V3v))) (ap (ap (ap (c_2Ebool_2ECOND \\ & ty_2Erealax_2Ereal) (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) \\ V3v) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) (ap (ap c_2Erealax_2Ereal_add \\ & (ap (ap c_2Ereal_2E_2F V0x) V1y)) (ap (c_2Emarker_2Eunint ty_2Erealax_2Ereal) \\ & (ap (ap c_2Ereal_2E_2F V2u) V3v)))) (ap (ap (ap (c_2Ebool_2ECOND \\ & ty_2Erealax_2Ereal) (ap (ap (c_2Emin_2E_3D ty_2Erealax_2Ereal) \\ V1y) V3v)) (ap (ap c_2Ereal_2E_2F (ap (ap c_2Erealax_2Ereal_add \\ & V0x) V2u)) V3v)) (ap (ap c_2Ereal_2E_2F (ap (ap c_2Erealax_2Ereal_add \\ & (ap (ap c_2Erealax_2Ereal_mul V0x) V3v)) (ap (ap c_2Erealax_2Ereal_mul \\ & V2u) V1y))) (ap (ap c_2Erealax_2Ereal_mul V1y) V3v)))))))))) \end{aligned}$$