

thm_2Ereal_2Ele__rat (TMUoRmXxwqYPN- bAkZcTFXmjakXMn24CCjJ9)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow P \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_COND$ to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Definition 9 We define $c_2Emarker_2Eunint$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.V0x$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{3}$$

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 16 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 19 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 21 We define $c_2Earithmic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT2) n)$

Definition 22 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT1) n)$

Definition 23 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Definition 24 We define $c_2Earithmic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (13)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (14)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (15)$$

Definition 25 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40) a)$

Let $c_2Erealax_2Etreall_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (16)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (17)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}} \quad (18)$$

Definition 26 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 27 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS\ T1)$

Let $c_2Erealax_2Etreall_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal} \quad (19)$$

Definition 28 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 29 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (20)$$

Definition 30 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 31 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (21)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ c_2Enum_2E0)\ V0n))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1n)\ V0m))))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ c_2Enum_2E0)\ V0m) = c_2Enum_2E0) \wedge \\ & ((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ V0m) = V0m) \wedge \\ & (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))\ V1n)) \wedge \\ & ((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ c_2Enum_2ESUC\ V1n)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))))))))) \quad (24) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V1n)) \wedge (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1n)\ V2p))) \Rightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V2p)))))) \quad (25) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p \ (ap \ (ap \ c_2Earithmetic_2E_3C_3D \\
& \quad (ap \ (ap \ c_2Earithmetic_2E_2B \ V0m) \ V1n)) \ (ap \ (ap \ c_2Earithmetic_2E_2B \\
& \quad V0m) \ V2p))) \Leftrightarrow (p \ (ap \ (ap \ c_2Earithmetic_2E_3C_3D \ V1n) \ V2p))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (\neg(V0m = V1n)) \Leftrightarrow ((p \ (ap \ (ap \ c_2Earithmetic_2E_3C_3D \ (ap \ c_2Enum_2ESUC \\
& \quad V0m)) \ V1n)) \vee (p \ (ap \ (ap \ c_2Earithmetic_2E_3C_3D \ (ap \ c_2Enum_2ESUC \\
& \quad V1n)) \ V0m))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap \ c_2Enum_2ESUC \ V0n) = (ap \ (ap \\
& \quad c_2Earithmetic_2E_2B \ (ap \ c_2Earithmetic_2ENUMERAL \ (ap \ c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \ V0n)))
\end{aligned} \tag{28}$$

Assume the following.

$$True \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2))))))
\end{aligned} \tag{30}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p \ V0t))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2. ((p \ V0t) \vee (\neg(p \ V0t)))) \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& \quad A_27a. (p \ V0t)) \Leftrightarrow (p \ V0t)))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& \quad (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& \quad (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& \quad (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{35}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \ V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \ V0t1) \ V1t2) = V1t2)))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Leftrightarrow ((p \ V0t) \Leftrightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge (((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\ & V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\ & V5y_27))))))))) \end{aligned} \quad (45)$$

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum. ((ap \\
& \quad (ap c_2Earithmic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum. (\forall V3m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2B \\
& \quad (ap c_2Earithmic_2ENUMERAL V2n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2A (\\
& \quad ap c_2Earithmic_2ENUMERAL V6n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum. (\forall V11m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2E_2D \\
& \quad (ap c_2Earithmic_2ENUMERAL V10n)) (ap c_2Earithmic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum. ((ap \\
& \quad (ap c_2Earithmic_2EEXP c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Earithmic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum. ((ap (ap c_2Earithmic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum. (\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmic_2EEXP (ap c_2Earithmic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmic_2ENUMERAL V16m)) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmic_2EEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& \quad c_2Earithmic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmic_2ENUMERAL V17n)) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmic_2ENUMERAL V18n)) = (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmic_2EZERO)))))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum. ((c_2Enum_2E0 = (ap c_2Earithmic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum. ((ap c_2Earithmic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum. (\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3E (ap c_2Earithmic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum. (\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3E (ap c_2Earithmic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V32n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D c_2Earithmic_2EZERO) \\
& \quad V32n)))))) \wedge ((\forall V33n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V33n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V33n)) c_2Enum_2E0)))))) \wedge ((\forall V34n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V34n)) c_2Enum_2E0)) \\
& \quad \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V34n)) c_2Enum_2E0)))))) \wedge \\
& \quad ((\forall V35n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V35n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V35n)) c_2Enum_2E0)))))) \wedge ((\forall V36n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V36n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V36n)) c_2Enum_2E0)))))) \wedge ((\forall V37n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V37n)) c_2Enum_2E0)) \\
& \quad \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V37n)) c_2Enum_2E0)))))) \wedge \\
& \quad ((\forall V38n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V38n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V38n)) c_2Enum_2E0)))))) \wedge ((\forall V39n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V39n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V39n)) c_2Enum_2E0)))))) \wedge ((\forall V40n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V40n)) c_2Enum_2E0)) \\
& \quad \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V40n)) c_2Enum_2E0)))))) \wedge \\
& \quad ((\forall V41n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V41n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D (ap c_2Earithmic_2ENUMERAL \\
& \quad V41n)) c_2Enum_2E0)))))) \wedge ((\forall V42n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmic_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V42n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C_3D \\
& \quad (ap c_2Earithmic_2ENUMERAL V42n)) c_2Enum_2E0)))))) \wedge ((\forall V43n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL V4$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m)))))))))) \\
& \tag{47}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V0x)))) \\
& \tag{48}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z)))))) \\
& \tag{49}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)))))) \\
& \tag{50}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V0m) V1n)))))) \\
& \tag{51}
\end{aligned}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Ereal_2Ereal_of_num\ V0m) = (ap\ c_2Ereal_2Ereal_of_num\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((\neg(V1y = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \Rightarrow ((ap\ (ap\ c_2Erealax_2Ereal_mul\ V1y)\ (ap\ (ap\ c_2Ereal_2E_2F\ V0x)\ V1y)) = V0x)))) \quad (53)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal. ((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V0x)) \Rightarrow ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y))\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V2z))) \Leftrightarrow (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V1y)\ V2z)))))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (p\ V1q) \Leftrightarrow (p\ V2r)) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (60)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{63}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2u \in ty_2Erealax_2Ereal. (\forall V3m \in ty_2Enum_2Enum. \\
& ((p \ (ap \ (ap \ c_2Ereal_2Ereal_lte \ (ap \ (ap \ c_2Ereal_2E_2F \ V0x) \ (ap \\
& c_2Ereal_2Ereal_of_num \ V1n))) \ (ap \ (ap \ c_2Ereal_2E_2F \ V2u) \ (\\
& ap \ c_2Ereal_2Ereal_of_num \ V3m)))) \Leftrightarrow (p \ (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \\
& 2) \ (ap \ (ap \ (c_2Emin_2E_3D \ ty_2Enum_2Enum) \ V1n) \ c_2Enum_2E0)) \\
& (ap \ (ap \ c_2Ereal_2Ereal_lte \ (ap \ (c_2Emarker_2Euint \ ty_2Erealax_2Ereal) \\
& (ap \ (ap \ c_2Ereal_2E_2F \ V0x) \ (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)))) \\
& (ap \ (ap \ c_2Ereal_2E_2F \ V2u) \ (ap \ c_2Ereal_2Ereal_of_num \ V3m)))) \\
& (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ 2) \ (ap \ (ap \ (c_2Emin_2E_3D \ ty_2Enum_2Enum) \\
& V3m) \ c_2Enum_2E0)) \ (ap \ (ap \ c_2Ereal_2Ereal_lte \ (ap \ (ap \ c_2Ereal_2E_2F \\
& V0x) \ (ap \ c_2Ereal_2Ereal_of_num \ V1n))) \ (ap \ (c_2Emarker_2Euint \\
& ty_2Erealax_2Ereal) \ (ap \ (ap \ c_2Ereal_2E_2F \ V2u) \ (ap \ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))) \ (ap \ (ap \ c_2Ereal_2Ereal_lte \ (ap \ (ap \ c_2Erealax_2Ereal_mul \\
& (ap \ c_2Ereal_2Ereal_of_num \ V3m)) \ V0x)) \ (ap \ (ap \ c_2Erealax_2Ereal_mul \\
& (ap \ c_2Ereal_2Ereal_of_num \ V1n) \ V2u)))))))))
\end{aligned}$$