

thm_2Ereal_2Epow_rat
 (TMU4FSrQmpHszKsq9D173oc3fRa8jLGwXff)

October 26, 2020

Let $c_2Enum_2ZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2ABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2ABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2ABS_num\ c_2Enum_2ZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A. \lambda P \in (2^{A-27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ P))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2ABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 6 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT2\ n)\ V)$

Let c_2 be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (7)$$

Definition 7 We define $c_2 \in \text{Emin_2E_3D_3D_3E}$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o} (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in$

Definition 11 We define $c_{\text{2Emin_2E_40}}$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (ap \ P \ x)) \ \text{then} \ (\lambda x. x \in A \wedge \text{of type } \iota \Rightarrow \iota)$.

Definition 12 We define c_Ebool_2ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Let c_2 be given. Assume the following.

c_2Earithmetic_2E_2A \in ((*ty_2Enum_2Enum*^{*ty_2Enum_2Enum*})

(8)

Definition 15. We define a 2Earithmatic 2EFIT1 to be $\lambda V0n \in t u$: 2Ema

Density = 16. Worked = 2E with anti-GENUMERAL rule. NVE = 54. 2E = 2E = VKE.

Let t , ν , E_{real} , E_{relax} be given. Assume the following.

Let $\nu g_{\text{ZEEreal}} \text{cal} \text{ZEEreal}$: ν be given. Assume the following.

$$nonempty \text{ } ty_ZERealx_ZEReal \quad (11)$$

Let `c_ZERealax_ZEReal-THET CLASS : t` be given. Assume the following.

$$c_{\text{2Erealax_2Ereal_REP_CLASS}} \in ((2^{(v_9 \rightarrow L_{par_2E} \wedge v_9 \rightarrow L_{par_cat} \wedge v_9 \rightarrow L_{par_cat} \wedge v_9 \rightarrow L_{par_cat})}) \wedge v_9 \rightarrow L_{par_cat}) \quad (12)$$

Definition 17 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)} \quad (13)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)} \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} \quad (15)$$

Definition 18 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 19 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} \quad (16)$$

Definition 20 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)} \quad (17)$$

Definition 21 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Definition 22 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (18)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (19)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal} \quad (20)$$

Assume the following.

$$\begin{aligned}
 & ((\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V0m) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V1m) (ap c_2Enum_2ESUC V2n)) = (ap (ap c_2Earithmetic_2E_2A V1m) (ap (ap c_2Earithmetic_2EEXP V1m) V2n)))))))
 \end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
 & (((p (ap c_2Earithmetic_2EODD c_2Enum_2E0)) \Leftrightarrow False) \wedge (\forall V0n \in ty_2Enum_2Enum.((p (ap c_2Earithmetic_2EODD (ap c_2Enum_2ESUC V0n))) \Leftrightarrow (\neg(p (ap c_2Earithmetic_2EODD V0n))))))
 \end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(
 ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))
 \end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(
 ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge
 (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge
 (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge
 (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge
 ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \wedge
 ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))
 \end{aligned} \tag{24}$$

Assume the following.

$$True \tag{25}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.((ap(ap(c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap(ap(ap(c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))) \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap(ap(ap(c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap(ap(ap(c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27))))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge \\
 & (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\
 & V1n))))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n)))))) \\
 & \quad (37)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(
 & (ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num \\
 & V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num \\
 & (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))) \\
 & \quad (38)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & ((\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Ereal_2Epow V0x) \\
 & c_2Enum_2E0) = (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge (\forall V1x \in \\
 & ty_2Erealax_2Ereal.(\forall V2n \in ty_2Enum_2Enum.((ap (ap c_2Ereal_2Epow \\
 & V1x) (ap c_2Enum_2ESUC V2n)) = (ap (ap c_2Erealax_2Ereal_mul V1x) \\
 & (ap (ap c_2Ereal_2Epow V1x) V2n)))))) \\
 & \quad (39)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
 & ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Erealax_2Ereal_neg \\
 & V1y)) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
 & V0x) V1y)))))) \\
 & \quad (40)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
 & ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) \\
 & V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
 & V0x) V1y)))))) \\
 & \quad (41)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Erealax_2Ereal_neg \\
 & (ap c_2Erealax_2Ereal_neg V0x)) = V0x)) \\
 & \quad (42)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
 & (\forall V2n \in ty_2Enum_2Enum.((ap (ap c_2Ereal_2Epow (ap (ap c_2Ereal_2E_2F \\
 & V0x) V1y)) V2n) = (ap (ap c_2Ereal_2E_2F (ap (ap c_2Ereal_2Epow V0x) \\
 & V2n)) (ap (ap c_2Ereal_2Epow V1y) V2n)))))) \\
 & \quad (43)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0x \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & (ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num V0x)) V1n) = \\
 & (ap c_2Ereal_2Ereal_of_num (ap (ap c_2Earithmetic_2EXP V0x) \\
 & V1n)))))) \\
 \end{aligned} \tag{44}$$

Theorem 1

$$\begin{aligned}
 & (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
 & (\forall V2a \in ty_2Enum_2Enum. (\forall V3y \in ty_2Erealax_2Ereal. \\
 & (((ap (ap c_2Ereal_2Epow V0x) c_2Enum_2E0) = (ap c_2Ereal_2Ereal_of_num \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2ZERO)))) \wedge \\
 & (((ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V1n))) = \\
 & (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \wedge (((ap (ap c_2Ereal_2Epow \\
 & (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT2 V1n))) = (ap c_2Ereal_2Ereal_of_num \\
 & c_2Enum_2E0)) \wedge (((ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num \\
 & (ap c_2Earithmetic_2ENUMERAL V2a))) (ap c_2Earithmetic_2ENUMERAL \\
 & V1n)) = (ap c_2Ereal_2Ereal_of_num (ap (ap c_2Earithmetic_2EXP \\
 & (ap c_2Earithmetic_2ENUMERAL V2a)) (ap c_2Earithmetic_2ENUMERAL \\
 & V1n))) \wedge (((ap (ap c_2Ereal_2Epow (ap c_2Erealax_2Ereal_neg \\
 & (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL V2a)))) \\
 & (ap c_2Earithmetic_2ENUMERAL V1n)) = (ap (ap (ap (c_2Ebool_2ECOND \\
 & (ty_2Erealax_2Ereal^ty_2Erealax_2Ereal)) (ap c_2Earithmetic_2EODD \\
 & (ap c_2Earithmetic_2ENUMERAL V1n))) c_2Erealax_2Ereal_neg) \\
 & (\lambda V4x \in ty_2Erealax_2Ereal. V4x)) (ap c_2Ereal_2Ereal_of_num \\
 & (ap (ap c_2Earithmetic_2EXP (ap c_2Earithmetic_2ENUMERAL V2a)) \\
 & (ap c_2Earithmetic_2ENUMERAL V1n)))) \wedge ((ap (ap c_2Ereal_2Epow \\
 & (ap (ap c_2Ereal_2E_2F V0x) V3y)) V1n) = (ap (ap c_2Ereal_2E_2F \\
 & (ap (ap c_2Ereal_2Epow V0x) V1n)) (ap (ap c_2Ereal_2Epow V3y) V1n)))))))) \\
 \end{aligned}$$