

thm_2Ereal_sigma_2EREAL_SUM_IMAGE_0 (TMFKgwoHieNhjxo7RV7oZ9AijkVo2Srymen)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V0t \in 2.V0t)) (\lambda V1t \in 2.V1t))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \tag{4}$$

Definition 7 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{5}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (7)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (8)$$

Definition 8 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge p\ of\ type\ \iota \Rightarrow \iota))$

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal\ a)))$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \quad (9)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \quad (11)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 11 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_neg\ (ty_2Erealax_2Ereal\ T1))$

Definition 12 We define c_2Ebool_2E2F5C to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.(c_2Ebool_2E21\ 2)\ t))))$

Let $c_2Epred_set_2Ecard : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Epred_set_2Ecard\ A.27a \in (ty_2Enum_2Enum)^{2^{A-27a}} \quad (12)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (13)$$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Definition 13 We define `c_2Erealax_2Ereal_mul` to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$
Let `c_2Erealax_2Etrealt_add` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (15)$$

Definition 14 We define `c_2Erealax_2Ereal_add` to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$
Let `c_2Epred_set_2EITSET` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27b})(2^{A_27a}))((A_27b^{A_27b})^{A_27a})) \quad (16)$$

Definition 15 We define `c_2Ereal_sigma_2EREAL_SUM_IMAGE` to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal$

Definition 16 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 17 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)((2^{A_27b})^{A_27a})) \quad (17)$$

Definition 18 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)$

Let `c_2Epred_set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})) \quad (18)$$

Definition 19 We define `c_2Epred_set_2EINSERT` to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)$

Definition 20 We define `c_2Epred_set_2EEMPTY` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 21 We define `c_2Epred_set_2EFINITE` to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a))$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ c_2Enum_2E0) = c_2Enum_2E0)) \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_2Enum_2Enum. (\forall V1b \in ty_2Enum_2Enum. (\\ & ((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num\ V0a))\ (ap\ c_2Ereal_2Ereal_of_num\ V1b)) = (ap\ c_2Ereal_2Ereal_of_num \\ & (ap\ (ap\ c_2Earithmetic_2E_2A\ V0a)\ V1b))) \wedge ((ap\ (ap\ c_2Erealax_2Ereal_mul \\ & (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V0a)) \\ & (ap\ c_2Ereal_2Ereal_of_num\ V1b)) = (ap\ c_2Erealax_2Ereal_neg \\ & (ap\ c_2Ereal_2Ereal_of_num\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0a) \\ & V1b)))) \wedge ((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num \\ & V0a))\ (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num \\ & V1b))) = (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num \\ & (ap\ (ap\ c_2Earithmetic_2E_2A\ V0a)\ V1b)))) \wedge ((ap\ (ap\ c_2Erealax_2Ereal_mul \\ & (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V0a)) \\ & (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V1b))) = \\ & (ap\ c_2Ereal_2Ereal_of_num\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0a) \\ & V1b)))))) \quad (27) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Ereal_2Ereal_of_num\ V0n) = (ap\ c_2Ereal_2Ereal_of_num \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num \\
& V0n)) = (ap\ c_2Ereal_2Ereal_of_num\ V1m)) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge \\
& (V1m = c_2Enum_2E0))) \wedge (((ap\ c_2Ereal_2Ereal_of_num\ V0n) = \\
& (ap\ c_2Erealax_2Ereal_neg\ (ap\ c_2Ereal_2Ereal_of_num\ V1m))) \Leftrightarrow \\
& ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap\ c_2Erealax_2Ereal_neg \\
& (ap\ c_2Ereal_2Ereal_of_num\ V0n)) = (ap\ c_2Erealax_2Ereal_neg \\
& (ap\ c_2Ereal_2Ereal_of_num\ V1m))) \Leftrightarrow (V0n = V1m)))))) \\
& \tag{28}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((p\ (ap \\
& (c_2Epred_set_2EFINITE\ A_27a)\ V0P)) \Rightarrow (\forall V1f \in (ty_2Erealax_2Ereal^{A_27a}). \\
& (\forall V2x \in ty_2Erealax_2Ereal. ((\forall V3y \in A_27a. ((ap\ V1f \\
& V3y) = V2x)) \Rightarrow ((ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE\ A_27a) \\
& V1f)\ V0P) = (ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num \\
& (ap\ (c_2Epred_set_2ECARD\ A_27a)\ V0P)))\ V2x))))))))) \\
& \tag{29}
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p\ (ap \\
& (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow ((ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE \\
& A_27a)\ (\lambda V1x \in A_27a. (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \\
& V0s) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))))
\end{aligned}$$