

thm_2Ereal_sigma_2EREAL_SUM_IMAGE_CROSS_SYM
(TMdFn-
VyN2e1YkY4VAu172stUdByePTqGvw9)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{3}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{4}$$

Definition 7 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27b})$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 A_27a) V0P)))$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (5)$$

Definition 11 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0P \in (2^{A_27a}). \lambda V1Q \in (2^{A_27b})$

Definition 12 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b)^{A_27c}. \lambda V1g \in (A_27c)^{A_27a}$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (6)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 13 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal \quad (9)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \quad (10)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (11)$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \quad (12)$$

Definition 14 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal. (ap (c_2Emin_2E_40 (c_2Erealx_2Ereal_REP_CLASS V0a)))$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (13)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (14)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \quad (15)$$

Definition 15 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 16 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET\ A_27a\ A_27b \in (((A_27b^{A_27b})(2^{A_27a}))^{((A_27b^{A_27b})^{A_27a})}) \quad (16)$$

Definition 17 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2Ereal)$

Definition 18 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 19 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Definition 21 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_5C_2F\ V0x\ s))$

Definition 22 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 23 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 24 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ V0s)$

Definition 25 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2) \Rightarrow (p \ V2t3)) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (27)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \quad \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in \\ & \quad A_{.27b}.(((ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \quad \forall V0x \in (ty_{.2Epair_2Eprod}\ A_{.27a}\ A_{.27b}).(\exists V1q \in A_{.27a}. \\ & \quad (\exists V2r \in A_{.27b}.(V0x = (ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b}) \\ & \quad V1q)\ V2r)))))) \\ & \hspace{15em} (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \quad \forall V0x \in (ty_{.2Epair_2Eprod}\ A_{.27a}\ A_{.27b}).((ap\ (ap\ (c_{.2Epair_2E_2C} \\ & \quad A_{.27a}\ A_{.27b})\ (ap\ (c_{.2Epair_2EFST}\ A_{.27a}\ A_{.27b})\ V0x))\ (ap\ (c_{.2Epair_2ESND} \\ & \quad A_{.27a}\ A_{.27b})\ V0x)) = V0x)) \\ & \hspace{15em} (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \quad \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.((ap\ (c_{.2Epair_2EFST}\ A_{.27a} \\ & \quad A_{.27b})\ (ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ V0x)\ V1y)) = V0x))) \\ & \hspace{15em} (32) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \quad \forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.((ap\ (c_{.2Epair_2ESND}\ A_{.27a} \\ & \quad A_{.27b})\ (ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ V0x)\ V1y)) = V1y))) \\ & \hspace{15em} (33) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\ & \quad nonempty\ A_{.27c} \Rightarrow (\forall V0f \in ((A_{.27c}^{A_{.27b}})^{A_{.27a}}).(\forall V1x \in \\ & \quad A_{.27a}.(\forall V2y \in A_{.27b}.((ap\ (ap\ (c_{.2Epair_2EUNCURRY}\ A_{.27a} \\ & \quad A_{.27b}\ A_{.27c})\ V0f)\ (ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ V1x)\ V2y)) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))))) \\ & \hspace{15em} (34) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ & \quad (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c_{.2Ebool_2EIN} \\ & \quad A_{.27a})\ V2x)\ V0s) \Leftrightarrow (p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V2x)\ V1t)))))) \\ & \hspace{15em} (35) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27b}). (\forall V2x \in \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b))\ V2x)\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27a\ A_27b) \\
& \quad V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ (ap\ (c_2Epair_2EFST \\
& \quad A_27a\ A_27b)\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ (c_2Epair_2ESND \\
& \quad A_27a\ A_27b)\ V2x))\ V1Q)))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27b}). (((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad A_27a)\ V0P)) \wedge (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V1Q))) \Rightarrow (p \\
& \quad (ap\ (c_2Epred_set_2EFINITE\ (ty_2Epair_2Eprod\ A_27a\ A_27b)) \\
& \quad (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27a\ A_27b)\ V0P)\ V1Q))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\
& \quad V0P)) \Rightarrow (\forall V1f_27 \in (A_27b^{A_27a}). ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EINJ \\
& \quad A_27a\ A_27b)\ V1f_27)\ V0P)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a \\
& \quad A_27b)\ V1f_27)\ V0P))) \Rightarrow (\forall V2f \in (ty_2Erealax_2Ereal^{A_27b}). \\
& \quad ((ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE\ A_27b)\ V2f)\ (ap \\
& \quad (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V1f_27)\ V0P)) = (ap\ (ap \\
& \quad (c_2Ereal_sigma_2EREAL_SUM_IMAGE\ A_27a)\ (ap\ (ap\ (c_2Ecombin_2Eo \\
& \quad A_27a\ ty_2Erealax_2Ereal\ A_27b)\ V2f)\ V1f_27))\ V0P))))))
\end{aligned} \tag{39}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (54)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\\ & \quad \forall V0f \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_{27a}\ A_{27b})}). \\ & (\forall V1s1 \in (2^{A_{27a}}).(\forall V2s2 \in (2^{A_{27b}}).(((p\ (ap\ (c_2Epred_set_2EFINITE \\ & \quad A_{27a})\ V1s1)) \wedge (p\ (ap\ (c_2Epred_set_2EFINITE\ A_{27b})\ V2s2))) \Rightarrow \\ & ((ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE\ (ty_2Epair_2Eprod \\ & \quad A_{27a}\ A_{27b}))\ (ap\ (c_2Epair_2EUNCURRY\ A_{27a}\ A_{27b}\ ty_2Erealax_2Ereal) \\ & \quad (\lambda V3x \in A_{27a}.(\lambda V4y \in A_{27b}.(ap\ V0f\ (ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_{27a}\ A_{27b})\ V3x)\ V4y))))))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_{27a} \\ & \quad A_{27b})\ V1s1)\ V2s2)) = (ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE \\ & \quad (ty_2Epair_2Eprod\ A_{27b}\ A_{27a}))\ (ap\ (c_2Epair_2EUNCURRY\ A_{27b} \\ & \quad A_{27a}\ ty_2Erealax_2Ereal)\ (\lambda V5y \in A_{27b}.(\lambda V6x \in A_{27a}.(\\ & \quad ap\ V0f\ (ap\ (ap\ (c_2Epair_2E_2C\ A_{27a}\ A_{27b})\ V6x)\ V5y))))))\ (ap\ (ap \\ & \quad (c_2Epred_set_2ECROSS\ A_{27b}\ A_{27a})\ V2s2)\ V1s1)))))) \end{aligned}$$