

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(3)

Definition 13 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E_5C_2F$

Definition 14 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (c_2Ebool_2E_21 2) V2t) V1t2) V0t1)$

Definition 16 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Ebool_2E_5C_2F$

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_5C_2F$

Definition 18 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2E_5C_2F$

Definition 19 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap (ap (c_2Ebool_2E_5C_2F$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(4)

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal$$
(5)

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal})$$
(6)

Definition 20 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal. (ap (c_2Emin_2E_40 (c_2Ebool_2E_7E$

Let $c_2Erealx_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}$$
(7)

Let $c_2Erealx_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}$$
(8)

Let $c_2Erealx_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_ABS_CLASS \in (ty_2Erealx_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}}$$
(9)

Definition 21 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty$

Definition 22 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 23 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{10}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{11}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{12}$$

Definition 24 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{13}$$

Let $c_2Epred_set_2EITSET : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EITSET \\ A_27a\ A_27b \in (((A_27b^{A_27b})^{(2^{A_27a})})^{((A_27b^{A_27b})^{A_27a})}) \end{aligned} \tag{14}$$

Definition 25 We define $c_2Ereal_sigma_2EREAL_SUM_IMAGE$ to be $\lambda A_27a : \iota.\lambda V0f \in (ty_2Erealax_2$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \tag{18}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (\\
& 2^{A_27a}).((\forall V2x \in A_27a.((p \ (ap \ V1P \ V2x)) \vee (p \ V0Q))) \Leftrightarrow ((\forall V3x \in \\
& A_27a.(p \ (ap \ V1P \ V3x))) \vee (p \ V0Q))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{28}$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \Rightarrow \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A.27a.(\forall V3x.27 \in A.27a.(\forall V4y \in A.27a. \\ & (\forall V5y.27 \in A.27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x.27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y.27)))) \Rightarrow ((ap (ap (ap (c.2Ebool.2ECOND A.27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c.2Ebool.2ECOND A.27a) V1Q) V3x.27) \\ & V5y.27))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p (ap (ap (c.2Ebool.2EIN A.27a) \\ & V2x) (ap (ap (c.2Epred._set.2EINTER A.27a) V0s) V1t))) \Leftrightarrow ((p (ap \\ & (ap (c.2Ebool.2EIN A.27a) V2x) V0s)) \wedge (p (ap (ap (c.2Ebool.2EIN \\ & A.27a) V2x) V1t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0s \in (2^{A.27a}).((ap (\\ & ap (c.2Epred._set.2EINTER A.27a) (c.2Epred._set.2EEMPTY A.27a)) \\ & V0s) = (c.2Epred._set.2EEMPTY A.27a))) \wedge (\forall V1s \in (2^{A.27a}). \\ & ((ap (ap (c.2Epred._set.2EINTER A.27a) V1s) (c.2Epred._set.2EEMPTY \\ & A.27a)) = (c.2Epred._set.2EEMPTY A.27a)))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1s \in \\ & (2^{A.27a}).(\forall V2t \in (2^{A.27a}).((ap (ap (c.2Epred._set.2EINTER \\ & A.27a) (ap (ap (c.2Epred._set.2EINSERT A.27a) V0x) V1s)) V2t) = \\ & (ap (ap (ap (c.2Ebool.2ECOND (2^{A.27a})) (ap (ap (c.2Ebool.2EIN \\ & A.27a) V0x) V2t)) (ap (ap (c.2Epred._set.2EINSERT A.27a) V0x) (\\ & ap (ap (c.2Epred._set.2EINTER A.27a) V1s) V2t))) (ap (ap (c.2Epred._set.2EINTER \\ & A.27a) V1s) V2t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1s \in \\ & (2^{A.27a}).((\neg(p (ap (ap (c.2Ebool.2EIN A.27a) V0x) V1s))) \Leftrightarrow ((ap \\ & (ap (c.2Epred._set.2EDELETE A.27a) V1s) V0x) = V1s)))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).((\\
& \quad (p\ (ap\ V0P\ (c_2Epred_set_2EEMPTY\ A_27a))) \wedge (\forall V1s \in (2^{A-27a}). \\
& \quad ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s)))) \Rightarrow \\
& \quad (\forall V2e \in A_27a.((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2e)\ V1s)))) \Rightarrow \\
& \quad (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2e)\ V1s)))))) \Rightarrow \\
& \quad (\forall V3s \in (2^{A-27a}).((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A-27a}).((p\ (ap \\
& \quad (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A-27a}). \\
& \quad (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINTER \\
& \quad \quad A_27a)\ V0s)\ V1t))))))
\end{aligned} \tag{36}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_add \\
(ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V0x) = V0x)) \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A-27a}). \\
& \quad (((ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE\ A_27a)\ V0f)\ (c_2Epred_set_2EEMPTY \\
& \quad \quad A_27a)) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \wedge (\forall V1e \in \\
& \quad \quad A_27a.(\forall V2s \in (2^{A-27a}).((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad \quad A_27a)\ V2s)) \Rightarrow ((ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE\ A_27a) \\
& \quad \quad V0f)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1e)\ V2s)) = (ap\ (ap \\
& \quad \quad c_2Erealax_2Ereal_add\ (ap\ V0f\ V1e))\ (ap\ (ap\ (c_2Ereal_sigma_2EREAL_SUM_IMAGE \\
& \quad \quad A_27a)\ V0f)\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V2s)\ V1e)))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \tag{41}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (49)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((p (ap \\ & (c.2Epred_set_2EFINITE A.27a) V0P)) \Rightarrow (\forall V1f \in (ty.2Erealax_2Ereal^{A.27a}). \\ & ((ap (ap (c.2Ereal_sigma_2EREAL_SUM_IMAGE A.27a) V1f) (ap \\ & (ap (c.2Epred_set_2EINTER A.27a) V0P) (\lambda V2p \in A.27a.(ap c.2Ebool_2E.7E \\ & (ap (ap (c.2Emin_2E_3D ty.2Erealax_2Ereal) (ap V1f V2p)) (ap c.2Ereal_2Ereal_of_num \\ & c.2Enum_2E0)))))) = (ap (ap (c.2Ereal_sigma_2EREAL_SUM_IMAGE \\ & A.27a) V1f) V0P)))) \end{aligned}$$