

thm_2Ereal_topology_2EAT_INFINITY (TMZ1Lcyp9y4TQeuRcR59Ew8o3mH1si5k4u9)

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Definition 1 We define `c_2Emin_2E3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E2` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E3D } (2^2)) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x))$

Definition 3 We define `c_2Emin_2E40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E3F` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } V 0P (\text{ap } (\text{c_2Emin_2E40 } A))))$

Let $\text{ty_2Ehreal_2Ehreal} : \iota$ be given. Assume the following.

$$\text{nonempty ty_2Ehreal_2Ehreal} \tag{1}$$

Let $\text{ty_2Epair_2Eprod} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 A 1) \tag{2}$$

Let $\text{ty_2Erealax_2Ereal} : \iota$ be given. Assume the following.

$$\text{nonempty ty_2Erealax_2Ereal} \tag{3}$$

Let $\text{c_2Erealax_2Ereal_REP_CLASS} : \iota$ be given. Assume the following.

$$\text{c_2Erealax_2Ereal_REP_CLASS} \in ((2^{(\text{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal}) \text{ty_2Erealax_2Ereal}})) \tag{4}$$

Definition 5 We define `c_2Ebool_2E21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E3D } (2^{A-27a})) (\lambda V 1x \in 2. V 1x)) (\lambda V 0x \in 2. V 0x))))$

Definition 6 We define `c_2Erealax_2Ereal_REP` to be $\lambda V 0a \in \text{ty_2Erealax_2Ereal}. (\text{ap } (\text{c_2Emin_2E40 } (\text{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal})) (\text{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal}))$

Let $\text{c_2Erealax_2Etrealt_lt} : \iota$ be given. Assume the following.

$$\text{c_2Erealax_2Etrealt_lt} \in ((2^{(\text{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal}) \text{ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal}})) \tag{5}$$

Definition 7 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 8 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 \ 2) (\lambda V0t \in 2.V0t)).$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E \ V0t) c_2Ebool_2E_7E))$

Definition 11 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal.$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty \ A0 \Rightarrow nonempty \ (ty_2Ereal_topology_2Enet \ A0) \quad (6)$$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow c_2Ereal_topology_2Enetord \ A_27a \in ((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet \ A_27a)} \quad (7)$$

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)}) \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)})} \quad (10)$$

Definition 12 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)$

Definition 13 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. (ap \ c_2Erealax_2Ereal_neg \ T1)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (11)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (13)$$

Definition 14 We define c_2Enum_2E0 to be $(ap \ c_2Enum_2EABS_num \ c_2Enum_2EZERO_REP).$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (14)$$

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 \ 2) (\lambda V2t \in$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 17 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Definition 18 We define $c_2Ereal_2Ereal_ge$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow c_2Ereal_topology_2Emk_net \\ & A_27a \in ((ty_2Ereal_topology_2Enet \ A_27a)^{(2^{A_27a}})^{A_27a}) \end{aligned} \quad (15)$$

Definition 19 We define $c_2Ereal_topology_2Eat_infinity$ to be $(ap (c_2Ereal_topology_2Emk_net \ ty_2E$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 \ 2) (\lambda V2t \in$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \\ & \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\neg(\forall V1x \in \\ A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\ 2.((\exists V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A_27a.(p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A_27a}).((\exists V2x \in A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A_27a.(p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (\\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ p\ V0A)) \vee (\neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ ((p\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V0x)\ V1y)) \vee (p\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\ V1y)\ V0x)))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (32)$$

Assume the following.

$$\begin{aligned} &(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ &(\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V2z))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} &\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ &(\forall V0a \in (ty_2Ereal_topology_2Enet\ A_27a).((ap (c_2Ereal_topology_2Emk_net \\ &\ A_27a) (ap (c_2Ereal_topology_2Enetord\ A_27a) V0a)) = V0a)) \wedge \\ &(\forall V1r \in ((2^{A_27b})^{A_27b}).((\forall V2x \in A_27b.(\forall V3y \in \\ &\ A_27b.((\forall V4z \in A_27b.((p (ap (ap V1r V4z) V2x)) \Rightarrow (p (ap (ap \\ &\ V1r V4z) V3y)))) \vee (\forall V5z \in A_27b.((p (ap (ap V1r V5z) V3y)) \Rightarrow \\ &\ (p (ap (ap V1r V5z) V2x))))))) \Leftrightarrow ((ap (c_2Ereal_topology_2Enetord \\ &\ A_27b) (ap (c_2Ereal_topology_2Emk_net\ A_27b) V1r)) = V1r)))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (36)$$

Assume the following.

$$\begin{aligned} &(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ &(((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} &(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ &((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (39)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ &(p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ &p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ &((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{44}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p \ (ap \ (ap \ (ap \ (c_2Ereal_topology_2Enetord \ ty_2Erealax_2Ereal) \\
& c_2Ereal_topology_2Eat_infinity) \ V0x) \ V1y)) \Leftrightarrow (p \ (ap \ (ap \ c_2Ereal_2Ereal_ge \\
& (ap \ c_2Ereal_2Eabs \ V0x)) \ (ap \ c_2Ereal_2Eabs \ V1y))))))
\end{aligned}$$