

thm_2Ereal__topology_2EBAIRE__ALT (TMMB- FiVTZu6zhAV7ZfUxLrHLUq4PHQAfzsL)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 \ A1) \tag{1}$$

Definition 4 We define `c_2Ebool_2EIN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `c_2Epair_2EABS__prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Epair_2EABS__prod } A_27a \ A_27b \in ((\text{ty_2Epair_2Eprod } A_27a \ A_27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 8 We define `c_2Epair_2E_2C` to be $\lambda A. 27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A. 27a. \lambda V1y \in A_27b. (\text{ap } (\text{c_2Ebool_2E_21 } 2))$

Let `c_2Epred__set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Epred__set_2EGSPEC } A_27a \ A_27b \in ((2^{A-27a})^{((\text{ty_2Epair_2Eprod } A_27a \ 2)^{A-27b})}) \tag{3}$$

Definition 9 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in ($

Definition 10 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ ($

Definition 11 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A-27a})}).(ap\ (c_2Epred_set_2EIMAGE\ ($

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ereal_topology_2Eball : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Eball \in ((2^{ty_2Erealax_2Ereal})(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)) \quad (5)$$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (6)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (7)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})ty_2Erealax_2Ereal) \quad (8)$$

Definition 12 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ($

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (9)$$

Definition 13 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP).$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (13)$$

Definition 15 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E2$

Definition 16 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 17 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 18 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E2F) (\lambda V0t \in 2.V0t))$.

Definition 19 We define c_2Ebool_2E3F to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E3D_3D_3E V0t) c_2Ebool_2E2F$

Definition 20 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 21 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_2Ereal_topo$

Definition 22 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2F)$.

Definition 23 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_s$

Definition 24 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in ($

Definition 25 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E2F) (\lambda V2t \in$

Definition 26 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 27 We define $c_2Ereal_topology_2Eclosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (ap (c_2Epred$

Definition 28 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (14)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (15)$$

Definition 29 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap (c_2Etopology_2Etopology ty_2Erealax$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Eopen_in A_27a \in ((2^{(2^{A_27a})})^{(ty_2Etopology_2Etopology A_27a)}) \quad (16)$$

Definition 30 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopo$

Definition 31 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p \ V0t))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p \ V0t) \Leftrightarrow (p \ V0t)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (29)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (V0x = V0x)) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\neg(\forall V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\neg(\exists V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a. (\neg(p (ap V0P V2x))))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). (((p V0P) \wedge (\forall V2x \in A.27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a. ((p V0P) \wedge (p (ap V1Q V3x))))) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). (((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). (((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (43)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (44)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (45)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1s \in (2^{A_{.27a}}).((p (ap (c_{.2E}pred_{.set}_{.2E}countable A_{.27a}) V1s)) \Rightarrow (p (ap (c_{.2E}pred_{.set}_{.2E}countable A_{.27b}) (ap (ap (c_{.2E}pred_{.set}_{.2E}IMAGE A_{.27a} A_{.27b}) V0f) V1s)))))) \quad (46)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (47)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V2x) V1t)))))) \quad (48)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2E}pair_{.2E}prod A_{.27a} 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E}IN A_{.27a}) V1v) (ap (c_{.2E}pred_{.set}_{.2E}EGSPEC A_{.27a} A_{.27b}) V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap (ap (c_{.2E}pair_{.2E}_{.2C} A_{.27a} 2) V1v) c_{.2E}bool_{.2E}ET) = (ap V0f V2x)))))) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0x) (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\ V2x) (ap (ap (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t)))) \Leftrightarrow ((p (ap \\ (ap (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p (ap (ap (c_2Ebool_2EIN \\ A_27a)\ V2x)\ V1t)))))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\ V2x) (ap (ap (c_2Epred_set_2EDIFF\ A_27a)\ V0s)\ V1t)))) \Leftrightarrow ((p (ap (\\ (ap (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (\neg (p (ap (ap (c_2Ebool_2EIN \\ A_27a)\ V2x)\ V1t)))))))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ ((p (ap (ap (c_2Ebool_2EIN\ A_27b)\ V0y) (ap (ap (c_2Epred_set_2EIMAGE \\ A_27a\ A_27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((ap (ap (c_2Epred_set_2EIMAGE\ A_27a\ A_27a) (\lambda V1x \in A_27a.V1x)) V0s) = V0s)) \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0P \in (2^{A_27a}). (\forall V1f \in (A_27a^{A_27b}). (\forall V2s \in \\ (2^{A_27b}). ((\forall V3y \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\ V3y) (ap (ap (c_2Epred_set_2EIMAGE\ A_27b\ A_27a)\ V1f)\ V2s)))) \Rightarrow (\\ p (ap\ V0P\ V3y)))) \Leftrightarrow (\forall V4x \in A_27b. ((p (ap (ap (c_2Ebool_2EIN \\ A_27b)\ V4x)\ V2s)) \Rightarrow (p (ap\ V0P\ (ap\ V1f\ V4x)))))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1sos \in \\ (2^{(2^{A_27a})}). ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0x) (ap (c_2Epred_set_2EBIGUNION \\ A_27a)\ V1sos)))) \Leftrightarrow (\exists V2s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN \\ A_27a)\ V0x)\ V2s)) \wedge (p (ap (ap (c_2Ebool_2EIN\ (2^{A_27a})\ V2s)\ V1sos)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((2^{A_27b})^{A_27a}).(\forall V1s \in (2^{A_27a}).((ap\ (\\
& \quad c_2Epred_set_2EBIGINTER\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A_27a\ (2^{A_27b}))\ V0f)\ V1s)) = (ap\ (c_2Epred_set_2EGSPEC\ A_27b \\
& \quad A_27b)\ (\lambda V2y \in A_27b.(ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ 2)\ V2y)\ (\\
& \quad ap\ (c_2Ebool_2E_21\ A_27a)\ (\lambda V3x \in A_27a.(ap\ (ap\ c_2Emin_2E_3D_3D_3E \\
& \quad (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27b)\ V2y)\ (ap\ V0f\ V3x)))))))))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0u \in (2^{A_27a}).(\forall V1s \in \\
& \quad (2^{(2^{A_27a})}).((ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ V0u)\ (ap \\
& \quad (c_2Epred_set_2EBIGINTER\ A_27a)\ V1s)) = (ap\ (c_2Epred_set_2EBIGUNION \\
& \quad A_27a)\ (ap\ (c_2Epred_set_2EGSPEC\ (2^{A_27a})\ (2^{A_27a}))\ (\lambda V2t \in \\
& \quad (2^{A_27a}).(ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27a})\ 2)\ (ap\ (ap\ (c_2Epred_set_2EDIFF \\
& \quad A_27a)\ V0u)\ V2t))\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2t)\ V1s)))))) \\
& \hspace{15em} (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A_27a}).(\forall V1f \in ((2^{A_27b})^{A_27a}).(\forall V2g \in \\
& \quad ((2^{A_27b})^{A_27a}).((\forall V3x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V3x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27b)\ (ap \\
& \quad V1f\ V3x))\ (ap\ V2g\ V3x)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27b) \\
& \quad (ap\ (c_2Epred_set_2EBIGUNION\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A_27a\ (2^{A_27b}))\ V1f)\ V0s)))\ (ap\ (c_2Epred_set_2EBIGUNION\ A_27b) \\
& \quad (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ (2^{A_27b}))\ V2g)\ V0s)))))) \\
& \hspace{15em} (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad ((ap\ c_2Ereal_topology_2EDist\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ V0x)\ V1y)) = (ap\ c_2Ereal_topology_2EDist \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad V1y)\ V0x)))) \\
& \hspace{15em} (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad (\forall V0s \in (2^{(2^{A-27a})}).(\forall V1t \in (2^{A-27a}).((ap\ (ap \\
& \quad (c.2Epred_set_2EINTER\ A.27a)\ (ap\ (c.2Epred_set_2EBIGUNION \\
& \quad A.27a)\ V0s))\ V1t) = (ap\ (c.2Epred_set_2EBIGUNION\ A.27a)\ (ap\ (c.2Epred_set_2EGSPEC \\
& \quad (2^{A-27a})\ (2^{A-27a}))\ (\lambda V2x \in (2^{A-27a}).(ap\ (ap\ (c.2Epair_2E_2C \\
& \quad (2^{A-27a})\ 2)\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ V2x)\ V1t))) \\
& \quad (ap\ (ap\ (c.2Ebool_2EIN\ (2^{A-27a}))\ V2x)\ V0s)))))) \wedge (\forall V3s \in \\
& \quad (2^{(2^{A-27b})}).(\forall V4t \in (2^{A-27b}).((ap\ (ap\ (c.2Epred_set_2EINTER \\
& \quad A.27b)\ V4t)\ (ap\ (c.2Epred_set_2EBIGUNION\ A.27b)\ V3s)) = (ap\ (c.2Epred_set_2EBIGUNION \\
& \quad A.27b)\ (ap\ (c.2Epred_set_2EGSPEC\ (2^{A-27b})\ (2^{A-27b}))\ (\lambda V5x \in \\
& \quad (2^{A-27b}).(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A-27b})\ 2)\ (ap\ (ap\ (c.2Epred_set_2EINTER \\
& \quad A.27b)\ V4t)\ V5x))\ (ap\ (ap\ (c.2Ebool_2EIN\ (2^{A-27b}))\ V5x)\ V3s))))))))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(p\ (ap\ (ap\ (c.2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal)\ (ap\ (ap\ (c.2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal)\ c.2Ereal_topology_2Euclidean)\ V0s))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad (\forall V2e \in ty_2Erealax_2Ereal.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ ty_2Erealax_2Ereal) \\
& \quad V1y)\ (ap\ c.2Ereal_topology_2Eball\ (ap\ (ap\ (c.2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ V0x)\ V2e)))) \Leftrightarrow (p\ (ap\ (ap\ c.2Erealax_2Ereal_lt \\
& \quad (ap\ c.2Ereal_topology_2EDist\ (ap\ (ap\ (c.2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& \quad ty_2Erealax_2Ereal)\ V0x)\ V1y)))\ V2e)))))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1e \in ty_2Erealax_2Ereal. \\
& \quad (p\ (ap\ c.2Ereal_topology_2EOpen\ (ap\ c.2Ereal_topology_2Eball \\
& \quad (ap\ (ap\ (c.2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad V0x)\ V1e)))))) \\
& \hspace{15em} (64)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0u \in (2^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& \quad ((p\ (ap\ c.2Ereal_topology_2EOpen\ V1s)) \Rightarrow (p\ (ap\ (ap\ (c.2Etopology_2Eopen_in \\
& \quad ty_2Erealax_2Ereal)\ (ap\ (ap\ (c.2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal)\ c.2Ereal_topology_2Euclidean)\ V0u)) \\
& \quad (ap\ (ap\ (c.2Epred_set_2EINTER\ ty_2Erealax_2Ereal)\ V0u)\ V1s)))))) \\
& \hspace{15em} (65)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0u \in (2^{ty_2Erealax_2Ereal}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap c_2Ereal_topology_2EClosed V1s)) \Rightarrow (p (ap (ap (c_2Etopology_2EClosed_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V0u)) \\
& \quad (ap (ap (c_2Epred_set_2EINTER ty_2Erealax_2Ereal) V0u) V1s))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V0x) (ap c_2Ereal_topology_2Eclosure \\
& \quad V1s))) \Leftrightarrow (\forall V2e \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad \quad (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0) V2e)) \Rightarrow (\exists V3y \in \\
& \quad \quad ty_2Erealax_2Ereal.((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad \quad V3y) V1s)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_topology_2EDist \\
& \quad \quad (ap (ap (c_2Epair_2E2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad \quad \quad V3y) V0x))) V2e)))))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(p (ap c_2Ereal_topology_2EClosed \\
& \quad (ap c_2Ereal_topology_2Eclosure V0s))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(p (ap (ap (c_2Epred_set_2ESUBSET \\
& \quad ty_2Erealax_2Ereal) V0s) (ap c_2Ereal_topology_2Eclosure V0s))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& ((ap c_2Ereal_topology_2Eclosure (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal)) = \\
& \quad (c_2Epred_set_2EEMPTY ty_2Erealax_2Ereal))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0g \in (2^{(2^{ty_2Erealax_2Ereal})}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap c_2Ereal_topology_2Elocally c_2Ereal_topology_2Ecompact) \\
& \quad V1s)) \wedge ((p (ap (c_2Epred_set_2Ecountable (2^{ty_2Erealax_2Ereal}) \\
& \quad V0g)) \wedge (\forall V2t \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Ebool_2EIN \\
& \quad (2^{ty_2Erealax_2Ereal}) V2t) V0g)) \Rightarrow ((p (ap (ap (c_2Etopology_2Eopen_in \\
& \quad \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad \quad \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V1s)) \\
& \quad \quad V2t)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad \quad V1s) (ap c_2Ereal_topology_2Eclosure V2t))))))))) \Rightarrow (p (ap (ap \\
& \quad (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V1s) (ap c_2Ereal_topology_2Eclosure \\
& \quad \quad (ap (c_2Epred_set_2EBIGINTER ty_2Erealax_2Ereal) V0g))))))
\end{aligned} \tag{71}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (74)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (75)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (79)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (80)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (82)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (86)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ A_27a).(\forall V1s \in (2^{A_27a}).(\forall V2t \in (2^{A_27a}).((p \\ (ap (ap (c_2Etopology_2Eopen_in \ A_27a) \ V0top) \ V1s)) \wedge (p (ap (ap \\ (c_2Etopology_2Eclosed_in \ A_27a) \ V0top) \ V2t))) \Rightarrow (p (ap (ap (c_2Etopology_2Eopen_in \\ A_27a) \ V0top) (ap (ap (c_2Epred_set_2EDIFF \ A_27a) \ V1s) \ V2t)))))))) \end{aligned} \quad (87)$$

Theorem 1

$$\begin{aligned} (\forall V0g \in (2^{(2^{ty_2Erealax_2Ereal})}).(\forall V1s \in (2^{ty_2Erealax_2Ereal}). \\ (((p (ap (ap (c_2Ereal_topology_2Elocally \ c_2Ereal_topology_2Ecompact) \\ V1s)) \wedge (\neg(V1s = (c_2Epred_set_2EEMPTY \ ty_2Erealax_2Ereal)))) \wedge \\ ((p (ap (c_2Epred_set_2Ecountable \ (2^{ty_2Erealax_2Ereal})) \\ V0g)) \wedge ((ap (c_2Epred_set_2EBIGUNION \ ty_2Erealax_2Ereal) \ V0g) = \\ V1s)))) \Rightarrow (\exists V2t \in (2^{ty_2Erealax_2Ereal}).(\exists V3u \in \\ (2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Ebool_2EIN \ (2^{ty_2Erealax_2Ereal})) \\ V2t) \ V0g)) \wedge ((p (ap (ap (c_2Etopology_2Eopen_in \ ty_2Erealax_2Ereal) \\ (ap (ap (c_2Ereal_topology_2Esubtopology \ ty_2Erealax_2Ereal) \\ c_2Ereal_topology_2Eeuclidean) \ V1s)) \ V3u)) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET \\ ty_2Erealax_2Ereal) \ V3u) (ap (c_2Ereal_topology_2Eclosure \ V2t)))))))))) \end{aligned}$$