

thm_2Ereal__topology_2EBALL__MIN__INTER
(TMY6DqvJD4gpQXVYhhafvJdetVQse1hhBPi)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1f \in 2.V1f V0x)))$

Definition 4 We define $c_2Emarker_2E_Abbrev$ to be $\lambda V0x \in 2.V0x$.

Definition 5 We define $c_2Ebool_2E_EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod A_27a A_27b))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (3)$$

Definition 9 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod A_27a A_27b))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (4)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (6)$$

Definition 10 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge P\ x)$) of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ap\ P\ x)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 13 We define c_2Ebool_2E21 to be $(ap\ (c_2Ebool_2E21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 14 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21))$

Definition 15 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21))))$

Definition 17 We define c_2Ereal_2Emin to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etreallneg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallneg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Let $c_2Erealax_2Etrealleq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealleq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (10)$$

Definition 18 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 19 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) (11)$$

Definition 20 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 21 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega (12)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum (13)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega\omega}) (14)$$

Definition 22 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) (15)$$

Definition 23 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONJ$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) (16)$$

Let $c_2Ereal_topology_2Eball : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2Eball \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) (17)$$

Assume the following.

$$True (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) (20)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in \\
& (2^{A-27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a) \\
& V2x)\ (ap\ (ap\ (c.2Epred_set.2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (21) \\
& (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27a)\ V2x)\ V1t))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealx.2Ereal. (\forall V1y \in ty.2Erealx.2Ereal. \\
& (\forall V2z \in ty.2Erealx.2Ereal. ((p\ (ap\ (ap\ c.2Erealx.2Ereal_lt \\
& V2z)\ (ap\ (ap\ c.2Ereal.2Emin\ V0x)\ V1y))) \Leftrightarrow ((p\ (ap\ (ap\ c.2Erealx.2Ereal_lt \\
& V2z)\ V0x)) \wedge (p\ (ap\ (ap\ c.2Erealx.2Ereal_lt\ V2z)\ V1y)))))) (22)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealx.2Ereal. (\forall V1y \in ty.2Erealx.2Ereal. \\
& ((ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\
& ty.2Erealx.2Ereal)\ V0x)\ V1y)) = (ap\ c.2Ereal.2Eabs\ (ap\ (ap\ c.2Ereal.2Ereal_sub \\
& V0x)\ V1y)))) (23)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty.2Erealx.2Ereal. (\forall V1y \in ty.2Erealx.2Ereal. \\
& (\forall V2e \in ty.2Erealx.2Ereal. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ ty.2Erealx.2Ereal) \\
& V1y)\ (ap\ c.2Ereal_topology.2Eball\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\
& ty.2Erealx.2Ereal)\ V0x)\ V2e)))) \Leftrightarrow (p\ (ap\ (ap\ c.2Erealx.2Ereal_lt \\
& (ap\ c.2Ereal_topology.2EDist\ (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal \\
& ty.2Erealx.2Ereal)\ V0x)\ V1y))\ V2e)))))) (24)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0a \in ty.2Erealx.2Ereal. (\forall V1r \in ty.2Erealx.2Ereal. \\
& (\forall V2s \in ty.2Erealx.2Ereal. ((ap\ c.2Ereal_topology.2Eball \\
& (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\
& V0a)\ (ap\ (ap\ c.2Ereal.2Emin\ V1r)\ V2s))) = (ap\ (ap\ (c.2Epred_set.2EINTER \\
& ty.2Erealx.2Ereal)\ (ap\ c.2Ereal_topology.2Eball\ (ap\ (ap\ (c.2Epair.2E.2C \\
& ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal)\ V0a)\ V1r)))\ (ap\ c.2Ereal_topology.2Eball \\
& (ap\ (ap\ (c.2Epair.2E.2C\ ty.2Erealx.2Ereal\ ty.2Erealx.2Ereal) \\
& V0a)\ V2s))))))
\end{aligned}$$