

# thm\_2Ereal\_\_topology\_2EBIGINTER\_\_GSPEC (TMch1Z8hCoRVHQ9veogkcTu63bwr8Dym2C)

October 26, 2020

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (1)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2ESND\ A.27a\ A.27b \in (A.27b)^{(ty\_2Epair\_2Eprod\ A.27a\ A.27b)} \quad (2)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2EFST\ A.27a\ A.27b \in (A.27a)^{(ty\_2Epair\_2Eprod\ A.27a\ A.27b)} \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A.27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

**Definition 4** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in ((A.27c)^{A.27b})$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2EABS\_prod\ A.27a\ A.27b \in ((ty\_2Epair\_2Eprod\ A.27a\ A.27b)^{((2^{A.27b})^{A.27a})}) \quad (4)$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then}$  (the  $(\lambda x.x \in A \wedge p$   
of type  $\iota \Rightarrow \iota$ ).

**Definition 9** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b})}) \end{aligned} \quad (5)$$

**Definition 10** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x))$

**Definition 11** We define  $c\_2Epred\_set\_2EBIGINTER$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set$

**Definition 12** We define  $c\_2Ebool\_2E2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 14** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg (p V0t)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (11)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in \\
& A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in \\
& A\_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p \ (ap \ V0P \ V2x))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\
& (2^{A\_27a}).((\exists V2x \in A\_27a.((p \ (ap \ V0P \ V2x)) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\
& ((\exists V3x \in A\_27a.(p \ (ap \ V0P \ V3x))) \vee (\exists V4x \in A\_27a.(p \ ( \\
& ap \ V1Q \ V4x))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in \\ 2.(((\exists V2x \in A.27a.(p\ (ap\ V0P\ V2x))) \vee (p\ V1Q))) \Leftrightarrow (\exists V3x \in \\ A.27a.((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in \\ 2.(((\exists V2x \in A.27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A.27a.(p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\ 2^{A-27a}).((\exists V2x \in A.27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A.27a.(p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in ( \\ 2^{A-27a}).((\forall V2x \in A.27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in \\ A.27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ( \\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg( \\ p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ 2.(((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((ap (ap (c\_2Epair\_2EUNCURRY\ A\_27a\ A\_27b\ A\_27c)\ V0f) (ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1x)\ V2y)) = (ap (ap\ V0f\ V1x)\ V2y)))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).((\exists V1p \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(p (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p\_1 \in A\_27a.(\exists V3p\_2 \in A\_27b.(p (ap\ V0P (ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2p\_1)\ V3p\_2))))))) \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V1v) (ap (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap (ap (c\_2Epair\_2E\_2C\ A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1B \in (2^{(2^{A\_27a})}).((p (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V0x) (ap (c\_2Epred\_set\_2EBIGINTER\ A\_27a)\ V1B))) \Leftrightarrow (\forall V2P \in (2^{A\_27a}).((p (ap (ap (c\_2Ebool\_2EIN\ (2^{A\_27a})\ V2P)\ V1B)) \Rightarrow (p (ap (ap (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V2P)))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (52)$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\
& \text{nonempty } A\_27c \Rightarrow \forall A\_27d.\text{nonempty } A\_27d \Rightarrow \forall A\_27e.\text{nonempty } \\
& A\_27e \Rightarrow \forall A\_27f.\text{nonempty } A\_27f \Rightarrow \forall A\_27g.\text{nonempty } A\_27g \Rightarrow \\
& \forall A\_27h.\text{nonempty } A\_27h \Rightarrow \forall A\_27i.\text{nonempty } A\_27i \Rightarrow ( \\
& (\forall V0P \in (2^{A\_27a}).(\forall V1f \in ((2^{A\_27b})^{A\_27a}).((ap \\
& (c\_2Epred\_set\_2EBIGINTER A\_27b) (ap (c\_2Epred\_set\_2EGSPEC \\
& (2^{A\_27b}) A\_27a) (\lambda V2x \in A\_27a.(ap (ap (c\_2Epair\_2E\_2C (2^{A\_27b}) \\
& 2) (ap V1f V2x)) (ap V0P V2x)))))) = (ap (c\_2Epred\_set\_2EGSPEC A\_27b \\
& A\_27b) (\lambda V3a \in A\_27b.(ap (ap (c\_2Epair\_2E\_2C A\_27b 2) V3a) ( \\
& ap (c\_2Ebool\_2E\_21 A\_27a) (\lambda V4x \in A\_27a.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E \\
& (ap V0P V4x)) (ap (ap (c\_2Ebool\_2EIN A\_27b) V3a) (ap V1f V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A\_27d})^{A\_27c}).(\forall V6f \in (((2^{A\_27e})^{A\_27d})^{A\_27c}). \\
& ((ap (c\_2Epred\_set\_2EBIGINTER A\_27e) (ap (c\_2Epred\_set\_2EGSPEC \\
& (2^{A\_27e}) (ty\_2Epair\_2Eprod A\_27c A\_27d)) (ap (c\_2Epair\_2EUNCURRY \\
& A\_27c A\_27d (ty\_2Epair\_2Eprod (2^{A\_27e}) 2)) (\lambda V7x \in A\_27c. \\
& (\lambda V8y \in A\_27d.(ap (ap (c\_2Epair\_2E\_2C (2^{A\_27e}) 2) (ap (ap V6f \\
& V7x) V8y)) (ap (ap V5P V7x) V8y)))))) = (ap (c\_2Epred\_set\_2EGSPEC \\
& A\_27e A\_27e) (\lambda V9a \in A\_27e.(ap (ap (c\_2Epair\_2E\_2C A\_27e 2) \\
& V9a) (ap (c\_2Ebool\_2E\_21 A\_27c) (\lambda V10x \in A\_27c.(ap (c\_2Ebool\_2E\_21 \\
& A\_27d) (\lambda V11y \in A\_27d.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E (ap (ap V5P \\
& V10x) V11y)) (ap (ap (c\_2Ebool\_2EIN A\_27e) V9a) (ap (ap V6f V10x) \\
& V11y)))))))))) \wedge (\forall V12P \in (((2^{A\_27h})^{A\_27g})^{A\_27f}). \\
& (\forall V13f \in (((((2^{A\_27i})^{A\_27h})^{A\_27g})^{A\_27f}).((ap (c\_2Epred\_set\_2EBIGINTER \\
& A\_27i) (ap (c\_2Epred\_set\_2EGSPEC (2^{A\_27i}) (ty\_2Epair\_2Eprod \\
& A\_27f (ty\_2Epair\_2Eprod A\_27g A\_27h)) (ap (c\_2Epair\_2EUNCURRY \\
& A\_27f (ty\_2Epair\_2Eprod A\_27g A\_27h) (ty\_2Epair\_2Eprod (2^{A\_27i}) \\
& 2)) (\lambda V14x \in A\_27f.(ap (c\_2Epair\_2EUNCURRY A\_27g A\_27h (ty\_2Epair\_2Eprod \\
& (2^{A\_27i}) 2)) (\lambda V15y \in A\_27g.(\lambda V16z \in A\_27h.(ap (ap (c\_2Epair\_2E\_2C \\
& (2^{A\_27i}) 2) (ap (ap (ap V13f V14x) V15y) V16z)) (ap (ap (ap V12P V14x) \\
& V15y) V16z)))))))))) = (ap (c\_2Epred\_set\_2EGSPEC A\_27i A\_27i) \\
& (\lambda V17a \in A\_27i.(ap (ap (c\_2Epair\_2E\_2C A\_27i 2) V17a) (ap (c\_2Ebool\_2E\_21 \\
& A\_27f) (\lambda V18x \in A\_27f.(ap (c\_2Ebool\_2E\_21 A\_27g) (\lambda V19y \in \\
& A\_27g.(ap (c\_2Ebool\_2E\_21 A\_27h) (\lambda V20z \in A\_27h.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E \\
& (ap (ap (ap V12P V18x) V19y) V20z)) (ap (ap (c\_2Ebool\_2EIN A\_27i) \\
& V17a) (ap (ap (ap V13f V18x) V19y) V20z))))))))))))))
\end{aligned}$$