

thm_2Ereal__topology_2EBIGUNION__CONNECTED__COMPONENTS

(TMd2v4ZVxiQvYxTY6wyx9CtvT8XbyT3jYxr)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (3)$$

Definition 7 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27b})$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ V0P))))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A-27b})^{A-27a}}) \quad (4)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Ebool_2E_21\ 2)\ (ap\ (c_2Ebool_2E_21\ 2)\ V1y)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A-27b}}) \quad (5)$$

Definition 12 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A-27a}). (ap\ V1f\ V0x)))$

Definition 13 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A-27a})}). (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ V0P))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (6)$$

Definition 14 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (ap\ (c_2Ebool_2E_21\ 2)\ V1t)))$

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2F)$.

Definition 16 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (ap\ (c_2Ebool_2E_21\ 2)\ V1t)))$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V2t))))$

Definition 18 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (ap\ (c_2Ebool_2E_21\ 2)\ V1t)))$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (7)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (8)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (9)$$

Definition 19 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E_40\ A_27a\ V0a))$

Let $c_2Erealx_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Definition 20 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (13)$$

Definition 21 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \quad (14)$$

Definition 22 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealx_2Ereal}).(ap\ c_2Ebool_2E2)$

Definition 23 We define $c_2Ereal_topology_2Econnected$ to be $\lambda V0s \in (2^{ty_2Erealx_2Ereal}).(ap\ c_2Ebool_2E2)$

Definition 24 We define $c_2Ereal_topology_2Econnected_component$ to be $\lambda V0s \in (2^{ty_2Erealx_2Ereal}).\lambda V$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow \\
& ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1a \in \\
& A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (\\
& ap V0P V1a))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\
& A.27b.(((ap (ap (c.2Epair.2E.2C A.27a A.27b) V0x) V1y) = (ap (ap \\
& (c.2Epair.2E.2C A.27a A.27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\
& \forall V0f \in ((ty.2Epair.2Eprod A.27a 2)^{A.27b}).(\forall V1v \in \\
& A.27a.((p (ap (ap (c.2Ebool.2EIN A.27a) V1v) (ap (c.2Epred.set.2EGSPEC \\
& A.27a A.27b) V0f))) \Leftrightarrow (\exists V2x \in A.27b.((ap (ap (c.2Epair.2E.2C \\
& A.27a 2) V1v) c.2Ebool.2ET) = (ap V0f V2x))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V0s)\ V1t)) \wedge \\ & (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V1t)\ V0s))) \Rightarrow (V0s = V1t)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0X \in (2^{A_27a}). (\forall V1P \in \\ & (2^{(2^{A_27a})}). ((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ (ap \\ & (c_2Epred_set_2EBIGUNION\ A_27a)\ V1P))\ V0X)) \Leftrightarrow (\forall V2Y \in (\\ & 2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2Y)\ V1P)) \Rightarrow (p\ (\\ & ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V2Y)\ V0X)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow \forall A_27e.nonempty \\ & A_27e \Rightarrow \forall A_27f.nonempty\ A_27f \Rightarrow \forall A_27g.nonempty\ A_27g \Rightarrow \\ & (\forall V0Q \in (2^{A_27b}). (\forall V1P \in (2^{A_27a}). (\forall V2f \in \\ & (A_27b^{A_27a}). ((\forall V3z \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ V3z)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27b\ A_27a)\ (\lambda V4x \in \\ & A_27a. (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x)))))) \Rightarrow \\ & (p\ (ap\ V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A_27a. ((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ V0Q \\ & (ap\ V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A_27d})^{A_27c}). (\forall V7f \in \\ & ((A_27b^{A_27d})^{A_27c}). ((\forall V8z \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ V8z)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27b\ (ty_2Epair_2Eprod \\ & A_27c\ A_27d))\ (ap\ (c_2Epair_2EUNCURRY\ A_27c\ A_27d\ (ty_2Epair_2Eprod \\ & A_27b\ 2))\ (\lambda V9x \in A_27c. (\lambda V10y \in A_27d. (ap\ (ap\ (c_2Epair_2E_2C \\ & A_27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))) \Rightarrow (p \\ & (ap\ V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A_27c. (\forall V12y \in A_27d. ((p \\ & (ap\ (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y)))))) \wedge \\ & (\forall V13P \in (((2^{A_27g})^{A_27f})^{A_27e}). (\forall V14f \in (((A_27b^{A_27g})^{A_27f})^{A_27e}). \\ & ((\forall V15z \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V15z)\ (\\ & ap\ (c_2Epred_set_2EGSPEC\ A_27b\ (ty_2Epair_2Eprod\ A_27e\ (ty_2Epair_2Eprod \\ & A_27f\ A_27g))\ (ap\ (c_2Epair_2EUNCURRY\ A_27e\ (ty_2Epair_2Eprod \\ & A_27f\ A_27g)\ (ty_2Epair_2Eprod\ A_27b\ 2))\ (\lambda V16w \in A_27e. (ap \\ & (c_2Epair_2EUNCURRY\ A_27f\ A_27g\ (ty_2Epair_2Eprod\ A_27b\ 2)) \\ & (\lambda V17x \in A_27f. (\lambda V18y \in A_27g. (ap\ (ap\ (c_2Epair_2E_2C\ A_27b \\ & 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x) \\ & V18y)))))) \Rightarrow (p\ (ap\ V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A_27e. (\forall V20x \in \\ & A_27f. (\forall V21y \in A_27g. ((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow \\ & (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow (\\
& (\forall V0P \in (2^{A.27a}).(\forall V1f \in ((2^{A.27b})^{A.27a}).((ap \\
& (c.2Epred_set_2EBIGUNION\ A.27b)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27b})\ A.27a)\ (\lambda V2x \in A.27a.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27b}) \\
& 2)\ (ap\ V1f\ V2x))\ (ap\ V0P\ V2x)))))) = (ap\ (c.2Epred_set_2EGSPEC\ A.27b \\
& A.27b)\ (\lambda V3a \in A.27b.(ap\ (ap\ (c.2Epair_2E_2C\ A.27b\ 2)\ V3a)\ (\\
& ap\ (c.2Ebool_2E_3F\ A.27a)\ (\lambda V4x \in A.27a.(ap\ (ap\ c.2Ebool_2E_2F_5C \\
& (ap\ V0P\ V4x))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27b)\ V3a)\ (ap\ V1f\ V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A.27d})^{A.27c}).(\forall V6f \in (((2^{A.27e})^{A.27d})^{A.27c}). \\
& ((ap\ (c.2Epred_set_2EBIGUNION\ A.27e)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27e})\ ty_2Epair_2Eprod\ A.27c\ A.27d))\ (ap\ (c.2Epair_2EUNCURRY \\
& A.27c\ A.27d\ (ty_2Epair_2Eprod\ (2^{A.27e})\ 2))\ (\lambda V7x \in A.27c. \\
& (\lambda V8y \in A.27d.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27e})\ 2)\ (ap\ (ap\ V6f \\
& V7x)\ V8y))\ (ap\ (ap\ V5P\ V7x)\ V8y)))))) = (ap\ (c.2Epred_set_2EGSPEC \\
& A.27e\ A.27e)\ (\lambda V9a \in A.27e.(ap\ (ap\ (c.2Epair_2E_2C\ A.27e\ 2) \\
& V9a)\ (ap\ (c.2Ebool_2E_3F\ A.27c)\ (\lambda V10x \in A.27c.(ap\ (c.2Ebool_2E_3F \\
& A.27d)\ (\lambda V11y \in A.27d.(ap\ (ap\ c.2Ebool_2E_2F_5C\ (ap\ (ap\ V5P\ V10x) \\
& V11y))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27e)\ V9a)\ (ap\ (ap\ V6f\ V10x)\ V11y)))))))))) \wedge \\
& (\forall V12P \in (((2^{A.27h})^{A.27g})^{A.27f}).(\forall V13f \in (((2^{A.27i})^{A.27h})^{A.27g})^{A.27f}). \\
& ((ap\ (c.2Epred_set_2EBIGUNION\ A.27i)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27i})\ ty_2Epair_2Eprod\ A.27f\ (ty_2Epair_2Eprod\ A.27g\ A.27h))) \\
& (ap\ (c.2Epair_2EUNCURRY\ A.27f\ (ty_2Epair_2Eprod\ A.27g\ A.27h) \\
& (ty_2Epair_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V14x \in A.27f.(ap\ (c.2Epair_2EUNCURRY \\
& A.27g\ A.27h\ (ty_2Epair_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V15y \in A.27g. \\
& (\lambda V16z \in A.27h.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27i})\ 2)\ (ap\ (ap \\
& (ap\ V13f\ V14x)\ V15y)\ V16z))\ (ap\ (ap\ (ap\ V12P\ V14x)\ V15y)\ V16z)))))) = \\
& (ap\ (c.2Epred_set_2EGSPEC\ A.27i\ A.27i)\ (\lambda V17a \in A.27i.(ap \\
& (ap\ (c.2Epair_2E_2C\ A.27i\ 2)\ V17a)\ (ap\ (c.2Ebool_2E_3F\ A.27f) \\
& (\lambda V18x \in A.27f.(ap\ (c.2Ebool_2E_3F\ A.27g)\ (\lambda V19y \in A.27g. \\
& (ap\ (c.2Ebool_2E_3F\ A.27h)\ (\lambda V20z \in A.27h.(ap\ (ap\ c.2Ebool_2E_2F_5C \\
& (ap\ (ap\ (ap\ V12P\ V18x)\ V19y)\ V20z))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27i) \\
& V17a)\ (ap\ (ap\ (ap\ V13f\ V18x)\ V19y)\ V20z)))))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p\ (ap\ (ap\ (ap\ c.2Ereal_topology_2Econnected_component\ V0s) \\
& V1x)\ V1x)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V1x) \\
& V0s))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (\mathcal{2}^{ty_2Erealax_2Ereal}). (\forall V1x \in ty_2Erealax_2Ereal. \\
& (p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap (ap \\
& c_2Ereal_topology_2Econnected_component V0s) V1x)) V0s)))) \quad (32)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (\mathcal{2}^{ty_2Erealax_2Ereal}). ((ap (c_2Epred_set_2EBIGUNION \\
& ty_2Erealax_2Ereal) (ap (c_2Epred_set_2EGSPEC (\mathcal{2}^{ty_2Erealax_2Ereal}) \\
& ty_2Erealax_2Ereal) (\lambda V1x \in ty_2Erealax_2Ereal. (ap (ap (c_2Epair_2E_2C \\
& (\mathcal{2}^{ty_2Erealax_2Ereal}) 2) (ap (ap c_2Ereal_topology_2Econnected_component \\
& V0s) V1x)) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V1x) V0s)))))) = \\
& V0s))
\end{aligned}$$