

thm_2Ereal_topology_2EBIGUNION_DIFF (TMKcZxC5tSaCaeeH4syYKtRiKmEnnmnJZRA)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2EIN to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.\lambda a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (V0P))))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda a.nonempty A \Rightarrow \forall A.\lambda b.nonempty A \Rightarrow c_2Epair_2EABS_prod A a A b \in ((ty_2Epair_2Eprod A a A b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0x \in A.\lambda V1y \in A.\lambda V2z \in A.(ap (c_2Epair_2EABS_prod A a A b) (V0x V1y V2z))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda a.nonempty A \Rightarrow \forall A.\lambda b.nonempty A \Rightarrow c_2Epred_set_2EGSPEC A a A b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A a A b)^{A-27b}}) \tag{3}$$

Definition 10 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (4)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (5)$$

Definition 11 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a}$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the $(\lambda x.x \in A \wedge$*
of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 14 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27a}). (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EDIFF\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1sos \in (2^{(2^{A_27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGUNION\ A_27a)\ V1sos))) \Leftrightarrow (\exists V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2s)\ V1sos)))))) \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow (\\
& (\forall V0P \in (2^{A.27a}).(\forall V1f \in ((2^{A.27b})^{A.27a}).((ap \\
& (c.2Epred_set_2EBIGUNION\ A.27b)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27b})\ A.27a)\ (\lambda V2x \in A.27a.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27b}) \\
& 2)\ (ap\ V1f\ V2x))\ (ap\ V0P\ V2x)))))) = (ap\ (c.2Epred_set_2EGSPEC\ A.27b \\
& A.27b)\ (\lambda V3a \in A.27b.(ap\ (ap\ (c.2Epair_2E_2C\ A.27b\ 2)\ V3a)\ (\\
& ap\ (c.2Ebool_2E_3F\ A.27a)\ (\lambda V4x \in A.27a.(ap\ (ap\ c.2Ebool_2E_2F_5C \\
& (ap\ V0P\ V4x))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27b)\ V3a)\ (ap\ V1f\ V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A.27d})^{A.27c}).(\forall V6f \in (((2^{A.27e})^{A.27d})^{A.27c}). \\
& ((ap\ (c.2Epred_set_2EBIGUNION\ A.27e)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27e})\ (ty_2Epair_2Eprod\ A.27c\ A.27d))\ (ap\ (c.2Epair_2EUNCURRY \\
& A.27c\ A.27d\ (ty_2Epair_2Eprod\ (2^{A.27e})\ 2))\ (\lambda V7x \in A.27c. \\
& (\lambda V8y \in A.27d.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27e})\ 2)\ (ap\ (ap\ V6f \\
& V7x)\ V8y))\ (ap\ (ap\ V5P\ V7x)\ V8y)))))) = (ap\ (c.2Epred_set_2EGSPEC \\
& A.27e\ A.27e)\ (\lambda V9a \in A.27e.(ap\ (ap\ (c.2Epair_2E_2C\ A.27e\ 2) \\
& V9a)\ (ap\ (c.2Ebool_2E_3F\ A.27c)\ (\lambda V10x \in A.27c.(ap\ (c.2Ebool_2E_3F \\
& A.27d)\ (\lambda V11y \in A.27d.(ap\ (ap\ c.2Ebool_2E_2F_5C\ (ap\ (ap\ V5P\ V10x) \\
& V11y))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27e)\ V9a)\ (ap\ (ap\ V6f\ V10x)\ V11y)))))))))) \wedge \\
& ((\forall V12P \in (((2^{A.27h})^{A.27g})^{A.27f}).(\forall V13f \in (((2^{A.27i})^{A.27h})^{A.27g})^{A.27f}). \\
& ((ap\ (c.2Epred_set_2EBIGUNION\ A.27i)\ (ap\ (c.2Epred_set_2EGSPEC \\
& (2^{A.27i})\ (ty_2Epair_2Eprod\ A.27f\ (ty_2Epair_2Eprod\ A.27g\ A.27h)) \\
& (ap\ (c.2Epair_2EUNCURRY\ A.27f\ (ty_2Epair_2Eprod\ A.27g\ A.27h) \\
& (ty_2Epair_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V14x \in A.27f.(ap\ (c.2Epair_2EUNCURRY \\
& A.27g\ A.27h\ (ty_2Epair_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V15y \in A.27g. \\
& (\lambda V16z \in A.27h.(ap\ (ap\ (c.2Epair_2E_2C\ (2^{A.27i})\ 2)\ (ap\ (ap \\
& (ap\ V13f\ V14x)\ V15y)\ V16z))\ (ap\ (ap\ (ap\ V12P\ V14x)\ V15y)\ V16z)))))) = \\
& (ap\ (c.2Epred_set_2EGSPEC\ A.27i\ A.27i)\ (\lambda V17a \in A.27i.(ap \\
& (ap\ (c.2Epair_2E_2C\ A.27i\ 2)\ V17a)\ (ap\ (c.2Ebool_2E_3F\ A.27f) \\
& (\lambda V18x \in A.27f.(ap\ (c.2Ebool_2E_3F\ A.27g)\ (\lambda V19y \in A.27g. \\
& (ap\ (c.2Ebool_2E_3F\ A.27h)\ (\lambda V20z \in A.27h.(ap\ (ap\ c.2Ebool_2E_2F_5C \\
& (ap\ (ap\ (ap\ V12P\ V18x)\ V19y)\ V20z))\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27i) \\
& V17a)\ (ap\ (ap\ (ap\ V13f\ V18x)\ V19y)\ V20z)))))))))))))
\end{aligned} \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{21}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{22}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (25)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (30)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (31)$$

Theorem 1

$$\forall A. 27a.nonempty \ A. 27a \Rightarrow (\forall V0s \in (2^{(2^{A-27a})}). (\forall V1t \in (2^{A-27a}). ((ap (ap (c. 2Epred_set. 2EDIFF \ A. 27a) (ap (c. 2Epred_set. 2EBIGUNION \ A. 27a) \ V0s)) \ V1t) = (ap (c. 2Epred_set. 2EBIGUNION \ A. 27a) (ap (c. 2Epred_set. 2EGSPEC \ (2^{A-27a}) \ (2^{A-27a})) (\lambda V2x \in (2^{A-27a}). (ap (ap (c. 2Epair. 2E \ 2C \ (2^{A-27a}) \ 2) (ap (ap (c. 2Epred_set. 2EDIFF \ A. 27a) \ V2x) \ V1t)) (ap (ap (c. 2Ebool. 2EIN \ (2^{A-27a}) \ V2x) \ V0s))))))))$$