

thm_2Ereal__topology_2EBILINEAR__BOUNDED (TMWAWQXdLggZ1xV55MW33kWZ7E4m5JP25Yg)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A a))))$

Definition 4 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (ap V0P (ap (c_2Emin_2E_40 A a))))))$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (5)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (6)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (7)$$

Definition 15 We define $c_2Eiterate_2E_2E_2E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (8)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (9)$$

Definition 16 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \quad (11)$$

Definition 19 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2E_2B))n$.

Definition 20 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (13)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})ty_2Erealax_2Ereal) \quad (14)$$

Definition 21 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40))a$.

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (16)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (17)$$

Definition 22 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal).r$.

Definition 23 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_neg)T1$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (18)$$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (19)$$

Definition 24 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$.

Definition 25 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$.

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 27 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Erealx_2Etrealmul_add : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealmul_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (20)$$

Definition 28 We define $c_2Erealx_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$.

Let $c_2Erealx_2Etrealmul_mul : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealmul_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (21)$$

Definition 29 We define $c_2Erealx_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$.

Definition 30 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a}$

Definition 31 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b})$.

Definition 32 We define $c_2Eiterate_2Eneutral$ to be $\lambda A_27a : \iota.\lambda V0op \in ((A_27a^{A_27a})^{A_27a}).(ap (c_2Emin$

Definition 33 We define $c_2Eiterate_2Esupport$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 34 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_$

Definition 35 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 36 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E21 (2$

Definition 37 We define $c_2Eiterate_2EITSET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in ((A_27a^{A_27a})^{A_27b}).\lambda V$

Definition 38 We define $c_2Eiterate_2Eiterate$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0op \in ((A_27b^{A_27b})^{A_27b}).\lambda V$

Definition 39 We define $c_2Eiterate_2ESum$ to be $\lambda A_27a : \iota.(ap (c_2Eiterate_2Eiterate A_27a ty_2Erealx_2Ereal$

Definition 40 We define $c_2Ereal_topology_2Elinear$ to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Erealx_2Ereal})$

Definition 41 We define $c_2Ereal_topology_2Ebilinear$ to be $\lambda V0f \in ((ty_2Erealx_2Ereal^{ty_2Erealx_2Ereal})$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p \ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(\\ & p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (\\ & (p \ V1B) \wedge (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (33)$$

Assume the following.

$$p (ap (c_2Epred_set_2EFINITE ty_2Enum_2Enum) (ap (ap c_2Eiterate_2E_2E_2E V0m) V1n)))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V2p) (ap (ap c_2Eiterate_2E_2E_2E V0m) V1n))) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V2p) V1n))))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V1c \in ty_2Erealax_2Ereal^{A_27a}). (\forall V2s \in (2^{A_27a}). ((ap (ap (c_2Eiterate_2ESum A_27a) V2s) (\lambda V3x \in A_27a.(ap (ap c_2Erealax_2Ereal_mul V1c) (ap V0f V3x)))) = (ap (ap c_2Erealax_2Ereal_mul V1c) (ap (ap c_2Eiterate_2ESum A_27a) V2s) V0f)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V1g \in (ty_2Erealax_2Ereal^{A_27a}). (\forall V2s \in (2^{A_27a}). (((p (ap (c_2Epred_set_2EFINITE A_27a) V2s)) \wedge (\forall V3x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN A_27a) V3x) V2s)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Eabs (ap V0f V3x)) (ap V1g V3x)))))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Eabs (ap (ap (c_2Eiterate_2ESum A_27a) V2s) V0f)) (ap (ap (c_2Eiterate_2ESum A_27a) V2s) V1g)))))) \quad (37)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1n \in ty_2Enum_2Enum.((ap (ap (c_2Eiterate_2ESum ty_2Enum_2Enum) (ap (ap c_2Eiterate_2E_2E_2E V1n) V1n)) V0f) = (ap V0f V1n)))) \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c.2Epair.2EFST\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c.2Epair.2ESND\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}). (\forall V1x \in \\ & A.27a. (\forall V2y \in A.27b. ((ap\ (ap\ (c.2Epair.2EUNCURRY\ A.27a \\ & A.27b\ A.27c)\ V0f)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27a\ A.27b)\ V1x)\ V2y))) = \\ & (ap\ (ap\ V0f\ V1x)\ V2y)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{(ty.2Epair.2Eprod\ A.27a\ A.27b)}). ((\forall V1p \in \\ & (ty.2Epair.2Eprod\ A.27a\ A.27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow (\forall V2p_{-1} \in \\ & A.27a. (\forall V3p_{-2} \in A.27b. (p\ (ap\ V0P\ (ap\ (ap\ (c.2Epair.2E.2C \\ & A.27a\ A.27b)\ V2p_{-1})\ V3p_{-2})))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27b}). (\forall V2x \in \\ & (ty.2Epair.2Eprod\ A.27a\ A.27b). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (ty.2Epair.2Eprod \\ & A.27a\ A.27b))\ V2x)\ (ap\ (ap\ (c.2Epred._set.2ECROSS\ A.27a\ A.27b) \\ & V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ (ap\ (c.2Epair.2EFST \\ & A.27a\ A.27b)\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ (ap\ (c.2Epair.2ESND \\ & A.27a\ A.27b)\ V2x))\ V1Q)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27b}). (((p\ (ap\ (c.2Epred._set.2EFINITE \\ & A.27a)\ V0P)) \wedge (p\ (ap\ (c.2Epred._set.2EFINITE\ A.27b)\ V1Q)))) \Rightarrow (p \\ & (ap\ (c.2Epred._set.2EFINITE\ (ty.2Epair.2Eprod\ A.27a\ A.27b)) \\ & (ap\ (ap\ (c.2Epred._set.2ECROSS\ A.27a\ A.27b)\ V0P)\ V1Q)))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealx.2Ereal. ((ap\ (ap\ c.2Erealx.2Ereal._add \\ & (ap\ c.2Erealx.2Ereal._neg\ V0x))\ V0x) = (ap\ c.2Ereal.2Ereal._of._num \\ & c.2Enum.2E0))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V1y) V0x))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad V0x) V0x)))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \Leftrightarrow ((p (ap (ap c_2Erealax_2Ereal_lt \\
& \quad V0x) V1y)) \vee (V0x = V1y))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x)))) \Leftrightarrow (V0x = V1y)))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) = \\
& (ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Eabs V0x) (ap c_2Ereal_2Eabs \\
& V1y))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x) \\
& V1y)) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V1y))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0y) V1x))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x) \\
& V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x) \\
& (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Erealax_2Ereal_neg V0x)) = V0x))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal}). \\
& (\forall V1c \in ty_2Erealax_2Ereal. (\forall V2x \in ty_2Erealax_2Ereal. \\
& (\forall V3y \in ty_2Erealax_2Ereal. ((p (ap c_2Ereal_topology_2Ebilinear \\
& V0h)) \Rightarrow ((ap (ap V0h (ap (ap c_2Erealax_2Ereal_mul V1c) V2x)) V3y) = \\
& (ap (ap c_2Erealax_2Ereal_mul V1c) (ap (ap V0h V2x) V3y))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal}). \\
& (\forall V1c \in ty_2Erealax_2Ereal. (\forall V2x \in ty_2Erealax_2Ereal. \\
& (\forall V3y \in ty_2Erealax_2Ereal. ((p (ap c_2Ereal_topology_2Ebilinear \\
& V0h)) \Rightarrow ((ap (ap V0h V2x) (ap (ap c_2Erealax_2Ereal_mul V1c) V3y)) = \\
& (ap (ap c_2Erealax_2Ereal_mul V1c) (ap (ap V0h V2x) V3y))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27b}). (\forall V2f \in \\
& \quad (ty_2Erealax_2Ereal^{A_27a}). (\forall V3g \in (ty_2Erealax_2Ereal^{A_27b}). \\
& \quad (\forall V4h \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{ty_2Erealax_2Ereal}). \\
& \quad (((p (ap c_2Ereal_topology_2Ebilinear V4h)) \wedge ((p (ap (c_2Epred_set_2EFINITE \\
& \quad A_27a) V0s)) \wedge (p (ap (c_2Epred_set_2EFINITE A_27b) V1t)))) \Rightarrow (\\
& \quad (ap (ap V4h (ap (ap (c_2Eiterate_2ESum A_27a) V0s) V2f)) (ap (ap (\\
& \quad c_2Eiterate_2ESum A_27b) V1t) V3g)) = (ap (ap (c_2Eiterate_2ESum \\
& \quad (ty_2Epair_2Eprod A_27a A_27b)) (ap (ap (c_2Epred_set_2ECROSS \\
& \quad A_27a A_27b) V0s) V1t)) (ap (c_2Epair_2EUNCURRY A_27a A_27b ty_2Erealax_2Ereal) \\
& \quad (\lambda V5i \in A_27a. (\lambda V6j \in A_27b. (ap (ap V4h (ap V2f V5i)) (ap V3g \\
& \quad V6j))))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{66}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{68}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (70)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (76)$$

Theorem 1

$$(\forall V0h \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})_{ty_2Erealax_2Ereal}). ((p (ap c_2Ereal_topology_2Ebilinear V0h)) \Rightarrow (\exists V1B \in ty_2Erealax_2Ereal. (\forall V2x \in ty_2Erealax_2Ereal. (\forall V3y \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Eabs (ap (ap V0h V2x) V3y))) (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Erealax_2Ereal_mul V1B) (ap c_2Ereal_2Eabs V2x))) (ap c_2Ereal_2Eabs V3y))))))))))$$