

# thm\_2Ereal\_\_topology\_2EBILINEAR\_\_SUM (TMPkdhNQTCTigZ6dbb2oaP9yNpfwVMtPSoa)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A.\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{2}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \tag{3}$$

**Definition 5** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

**Definition 7** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ **if**  $(\exists x \in A.p (ap P x))$  **then** *(the  $(\lambda x.x \in A \wedge p (ap P x))$  of type  $\iota \Rightarrow \iota$ ).*

**Definition 9** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) (\lambda V1x \in 2.V1x))))$

**Definition 10** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

**Definition 11** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (ap\ V2t\ t1))))$   
Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}})$$
(4)

**Definition 13** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Epair\_2E\_21\ 2)\ (x\ y)))$   
Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}})$$
(5)

**Definition 14** We define  $c\_2Epred\_set\_2ECROSS$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0P \in (2^{A\_27a}). \lambda V1Q \in (2^{A\_27b}). (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b)\ (P\ Q)))$

**Definition 15** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1g \in (A\_27c^{A\_27a}). (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b\ A\_27c)\ (f\ g)))$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal$$
(6)

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal$$
(7)

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal})$$
(8)

**Definition 16** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Emin\_2E\_40\ 2)\ (a\ a))$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})$$
(9)

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})$$
(10)

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}}$$
(11)

**Definition 17** We define  $c\_Erealax\_Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 18** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 19** We define  $c\_Eiterate\_Eneutral$  to be  $\lambda A\_27a : \iota.\lambda V0op \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (c\_2Emin\_2Emin))$

**Definition 20** We define  $c\_Ebool\_2E2E$  to be  $(ap\ (c\_2Ebool\_2E2E\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 21** We define  $c\_Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t))\ c\_2Ebool\_2E7E)$

**Definition 22** We define  $c\_Eiterate\_2Esupport$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V0f$

**Definition 23** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ (c\_2Ebool\_2E2E\ 2))\ V1t1))\ V2t2))$

**Definition 24** We define  $c\_Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E2E\ 2))\ (\lambda V2t \in 2.V2t))\ V1t2))\ V0t1$

**Definition 25** We define  $c\_Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E2E\ 2))\ V1s)$

**Definition 26** We define  $c\_Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E2E)$ .

**Definition 27** We define  $c\_Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E2E\ 2))\ V0s)$

**Definition 28** We define  $c\_Eiterate\_2EITSET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27b}).\lambda V0g$

**Definition 29** We define  $c\_Eiterate\_2Eiterate$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0op \in ((A\_27b^{A\_27b})^{A\_27b}).\lambda V0f$

**Definition 30** We define  $c\_Eiterate\_2ESum$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eiterate\_2Eiterate\ A\_27a\ ty\_2Erealax\_2Ereal))$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)} \quad (12)$$

**Definition 31** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 32** We define  $c\_2Ereal\_topology\_2Elinear$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$

**Definition 33** We define  $c\_2Ereal\_topology\_2Ebilinear$  to be  $\lambda V0f \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})^{ty\_2Erealax\_2Ereal})$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & 2. (((\forall V2x \in A\_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\ & A\_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}). (((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b\ A\_27b)\ (c\_2Ecombin\_2EI\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (c\_2Ecombin\_2EI\ A\_27a)) = V0f))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0s \in (2^{A\_27a}). (\forall V1t \in ((2^{A\_27b})^{A\_27a}). (\forall V2x \in \\ ((ty\_2Erealx\_2Ereal^{A\_27b})^{A\_27a}). (((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V0s)) \wedge (\forall V3i \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3i)\ V0s)) \Rightarrow (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27b)\ (ap\ V1t\ V3i)))))) \Rightarrow \\ ((ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27a)\ V0s)\ (\lambda V4i \in A\_27a. (ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27b)\ (ap\ V1t\ V4i))\ (ap\ V2x\ V4i)))) = (ap\ (ap\ (c\_2Eiterate\_2ESum\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (c\_2Epred\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27a\ A\_27b\ (ty\_2Epair\_2Eprod\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ 2))\ (\lambda V5i \in A\_27a. (\lambda V6j \in A\_27b. (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ 2)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V5i)\ V6j))\ (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V5i)\ V0s))\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V6j)\ (ap\ V1t\ V5i)))))))))) \\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27a\ A\_27b\ ty\_2Erealx\_2Ereal)\ (\lambda V7i \in A\_27a. (\lambda V8j \in A\_27b. (ap\ (ap\ V2x\ V7i)\ V8j)))))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in A\_27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & \quad nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\ & \quad A\_27a. (\forall V2y \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27a \\ & \quad A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V1x)\ V2y))) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0P \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}). ((\exists V1p \in \\ & \quad (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p\_1 \in \\ & \quad A\_27a. (\exists V3p\_2 \in A\_27b. (p\ (ap\ V0P\ (ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27a\ A\_27b)\ V2p\_1)\ V3p\_2))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & \quad (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}). (\forall V1v \in \\ & \quad A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & \quad A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & \quad A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \\ & \quad (\forall V1g \in (ty\_2Erealax\_2Ereal^{A\_27a}). (\forall V2s \in (2^{A\_27a}). \\ & \quad (((p\ (ap\ c\_2Ereal\_topology\_2Elinear\ V0f)) \wedge (p\ (ap\ (c\_2Epred\_set\_2EFINITE \\ & \quad A\_27a)\ V2s))) \Rightarrow ((ap\ V0f\ (ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27a)\ V2s)\ V1g)) = \\ & \quad (ap\ (ap\ (c\_2Eiterate\_2ESum\ A\_27a)\ V2s)\ (ap\ (ap\ (c\_2Ecombin\_2Eo \\ & \quad A\_27a\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0f)\ V1g)))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (45)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \quad \forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27b}).(\forall V2f \in \\ & \quad (ty\_2Erealax\_2Ereal^{A\_27a}).(\forall V3g \in (ty\_2Erealax\_2Ereal^{A\_27b}). \\ & \quad (\forall V4h \in ((ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal} ty\_2Erealax\_2Ereal). \\ & \quad (((p (ap c\_2Ereal\_topology\_2Ebilinear V4h)) \wedge ((p (ap (c\_2Epred\_set\_2EFINITE \\ & \quad A\_27a) V0s)) \wedge (p (ap (c\_2Epred\_set\_2EFINITE A\_27b) V1t)))) \Rightarrow ( \\ & \quad (ap (ap V4h (ap (ap (c\_2Eiterate\_2ESum A\_27a) V0s) V2f)) (ap (ap ( \\ & \quad c\_2Eiterate\_2ESum A\_27b) V1t) V3g)) = (ap (ap (c\_2Eiterate\_2ESum \\ & \quad (ty\_2Epair\_2Eprod A\_27a A\_27b)) (ap (ap (c\_2Epred\_set\_2ECROSS \\ & \quad A\_27a A\_27b) V0s) V1t)) (ap (c\_2Epair\_2EUNCURRY A\_27a A\_27b ty\_2Erealax\_2Ereal) \\ & \quad (\lambda V5i \in A\_27a.(\lambda V6j \in A\_27b.(ap (ap V4h (ap V2f V5i)) (ap V3g \\ & \quad V6j)))))))))) \end{aligned}$$