

# thm\_2Ereal\_\_topology\_2EBOUNDED\_\_CLOSED\_\_NEST (TMbf8AjnoZ8ew1GSJD3BFnn7DsKpR5pH4v9)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ecombin\_2E\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 4** We define  $c\_2Ecombin\_2E\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ecombin\_2E\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 6** We define  $c\_2Ebool\_2E\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) P)))$

**Definition 7** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_2Epred\_set\_2E\_2EMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2EF)$ .

**Definition 9** We define  $c\_2Ebool\_2E\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Epred\_set\_2E\_2SUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c\_2Emin\_2E\_3D\_3D\_3E s t))$

**Definition 12** We define  $c\_2Ebool\_2E\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 13** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (3)$$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)}) \quad (4)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (5)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (6)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (7)$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (10)$$

**Definition 15** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge p\ x) \text{ of type } \iota \Rightarrow \iota)$ .

**Definition 16** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (the (\lambda x.x \in A \wedge p\ x) \text{ of type } \iota \Rightarrow \iota)))$

Let  $c\_2Erealax\_2Etrealt\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealt\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 18** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t) c\_2Ebool\_2E\_7E))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (13)$$

**Definition 19** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

**Definition 20** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 21** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 22** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 23** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 24** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ereal\_topology\_2Enet : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ereal\_topology\_2Enet\ A0) \quad (14)$$

Let  $c\_2Ereal\_topology\_2Emk\_net : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} &\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Emk\_net \\ &A\_27a \in ((ty\_2Ereal\_topology\_2Enet\ A\_27a)^{(2^{A\_27a})^{A\_27a}}) \end{aligned} \quad (15)$$

**Definition 25** We define  $c\_2Ereal\_topology\_2Esequentially$  to be  $(ap\ (c\_2Ereal\_topology\_2Emk\_net\ ty\_2Ereal\_topology\_2Esequentially$

**Definition 26** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1g \in (A\_27c^{A\_27a})$

Let  $c\_2Ereal\_topology\_2Enetord : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ereal\_topology\_2Enetord\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Ereal\_topology\_2Enet\ A\_27a)}) \quad (16)$$

**Definition 27** We define  $c\_2Ereal\_topology\_2Etrivial\_limit$  to be  $\lambda A\_27a : \iota. \lambda V0net \in (ty\_2Ereal\_topology\_2Etrivial\_limit$

**Definition 28** We define  $c\_2Ereal\_topology\_2Eeventually$  to be  $\lambda A\_27a : \iota. \lambda V0p \in (2^{A\_27a}). \lambda V1net \in (ty\_2Ereal\_topology\_2Eeventually$

**Definition 29** We define  $c\_2Ereal\_topology\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota. \lambda V0f \in (ty\_2Erealax\_2Ereal^{A\_27a})$

**Definition 30** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 31** We define  $c\_2Ereal\_topology\_2Ecompact$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap\ (c\_2Ebool\_2E\_21\ 2)$

**Definition 32** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2E\_21)$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \end{aligned} \quad (17)$$

**Definition 33** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2E$

**Definition 34** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap\ (c\_2Ebool\_2E$

**Definition 35** We define  $c\_2Ereal\_topology\_2EClosed$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap\ c\_2Ereal\_topo$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \end{aligned} \quad (18)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (19)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (20)$$

**Definition 36** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 37** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. (ap\ c\_2Erealax\_2Ereal$

**Definition 38** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 39** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 40** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

**Definition 41** We define  $c\_2Ereal\_topology\_2Ebounded\_def$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap\ (c\_2Ebo$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty\_2Enum\_2Enum. (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\ c\_2Enum\_2E0\ V0n)))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ \forall V2p \in ty\_2Enum\_2Enum. (((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D \\ V0m\ V1n)) \wedge (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1n\ V2p)))) \Rightarrow (p\ ( \\ ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0m\ V2p))))))) \end{aligned} \quad (22)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmic\_2E\_3C\_3D V0m) V0m))) \quad (23)$$

Assume the following.

$$True \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))) \quad (30)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p \ V0P) \wedge (\forall V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))) \quad (37)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.(((\exists V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \vee (p \ V1Q)) \Leftrightarrow (\exists V3x \in A.27a.((p \ (ap \ V0P \ V3x)) \vee (p \ V1Q)))))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p \ V0P) \vee (\exists V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V3x)))))) \quad (39)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.(((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p \ (ap \ V0P \ V3x))) \wedge (p \ V1Q)))))) \quad (40)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((\forall V2x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \ V0P) \vee (\forall V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))) \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (45)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (46)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0P \in ((2^{A_{.27b}})^{A_{.27a}}).((\forall V1x \in A_{.27a}.(\exists V2y \in A_{.27b}.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_{.27b}^{A_{.27a}}).(\forall V4x \in A_{.27a}.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \quad (47)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ V0m)\ V1n)) \vee (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D\ V1n)\ V0m)))))) \quad (48)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}.nonempty\ A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27c}}).(\forall V2x \in A_{.27c}.((ap\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A_{.27c}\ A_{.27b}\ A_{.27a})\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \quad (49)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap\ (c\_2Ecombin\_2EI\ A_{.27a})\ V0x) = V0x)) \quad (50)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((\exists V1x \in A_{.27a}.(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A_{.27a})\ V1x)\ V0s))) \Leftrightarrow (\neg(V0s = (c\_2Epred\_set\_2EEMPTY\ A_{.27a})))))) \quad (51)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1l \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap (ap (ap (c\_2Ereal\_topology\_2E\_2D\_2D\_3E \\
& ty\_2Enum\_2Enum) V0s) V1l) c\_2Ereal\_topology\_2Esequentially)) \Leftrightarrow \\
& (\forall V2e \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V2e)) \Rightarrow (\exists V3N \in \\
& ty\_2Enum\_2Enum.(\forall V4n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V3N) V4n)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_topology\_2EDist \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& (ap V0s V4n)) V1l))) V2e))))))))) \\
& \hspace{15em} (52)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1s \in (2^{ty\_2Erealax\_2Ereal}). \\
& ((p (ap c\_2Ereal\_topology\_2EClosed V1s)) \Rightarrow ((\forall V2e \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V2e)) \Rightarrow (\exists V3y \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V3y) V1s)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_topology\_2EDist (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V3y) V0x))) V2e)))))) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN \\
& ty\_2Erealax\_2Ereal) V0x) V1s)))))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0r \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}).((\forall V1m \in \\
& ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V1m) V2n)) \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap V0r V1m)) (ap V0r V2n)))))) \Rightarrow \\
& (\forall V3n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V3n) (ap V0r V3n)))))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty\_2Erealax\_2Ereal}).(((p (ap c\_2Ereal\_topology\_2Ebunded\_def \\
& V0s)) \wedge (p (ap c\_2Ereal\_topology\_2EClosed V0s))) \Rightarrow (p (ap c\_2Ereal\_topology\_2Ecompact \\
& V0s)))) \\
& \hspace{15em} (55)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \hspace{10em} (56)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \hspace{10em} (57)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
& \hspace{15em} (58)
\end{aligned}$$



Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (61) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (62) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (63) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (64) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (65) \end{aligned}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (66)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (67)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0s \in ((2^{ty\_2Erealax\_2Ereal})^{ty\_2Enum\_2Enum}).((\forall V1n \in \\ & ty\_2Enum\_2Enum.(p (ap c\_2Ereal\_topology\_2EClosed (ap V0s V1n)))))) \wedge \\ & ((\forall V2n \in ty\_2Enum\_2Enum.(\neg((ap V0s V2n) = (c\_2Epred\_set\_2EEMPTY \\ & ty\_2Erealax\_2Ereal)))) \wedge ((\forall V3m \in ty\_2Enum\_2Enum.(\forall V4n \in \\ & ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V3m) V4n)) \Rightarrow \\ & (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap V0s \\ & V4n)) (ap V0s V3m)))))) \wedge (p (ap c\_2Ereal\_topology\_2Ebounded\_def \\ & (ap V0s c\_2Enum\_2E0)))))) \Rightarrow (\exists V5a \in ty\_2Erealax\_2Ereal. \\ & (\forall V6n \in ty\_2Enum\_2Enum.(p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\ & V5a) (ap V0s V6n)))))) \end{aligned}$$