

thm_2Ereal__topology_2ECARD__EQ__OPEN
(TMSvZaFQ8mJtDkmLhYpwGPBU4iPyr8oSZmq)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) P)))$

Definition 10 We define $c_2Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^{A_27b})$

Definition 11 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Emin_2E_40 A_27a) (ap V1t s))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal)}) \quad (7)$$

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 16 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E_2$

Definition 17 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2ET)$.

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (12)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (13)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (14)$$

Let $c_2Elist_2EHD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EHD\ A_27a \in (A_27a)^{(ty_2Elist_2Elist\ A_27a)} \quad (15)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (16)$$

Definition 18 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Definition 20 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (17)$$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (18)$$

Definition 21 We define $c_2Ereal_topology_2ECLOSED_interval$ to be $\lambda V0l \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))$

Definition 22 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (2^{A_27a})^{A_27b}$

Definition 23 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b)^{A_27a}.\lambda V1s \in (2^{A_27a})^{A_27b}$

Definition 24 We define $c_2Ecardinal_2Ecardeq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a})^{A_27b}.\lambda V1s2 \in (2^{A_27b})^{A_27a}$

Definition 25 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Let $c_2Ereal_topology_2EOPEN_interval : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EOPEN_interval \in ((2^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (19)$$

Definition 26 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (25)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (29)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{27})) \wedge ((p \ V1x_{27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{27})))))) \Rightarrow ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{27}) \Rightarrow (p \ V3y_{27})))))) \Rightarrow (30)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1Q \in 2.(((\exists V2x \in A_{27a}.(p \ (ap \ V0P \ V2x))) \Rightarrow (p \ V1Q)) \Leftrightarrow (\forall V3x \in A_{27a}.((p \ (ap \ V0P \ V3x)) \Rightarrow (p \ V1Q)))))) \Rightarrow (31)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow \forall A_{27c}.nonempty \ A_{27c} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in (2^{A_{27b}}).(\forall V2u \in (2^{A_{27c}}).(((p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardleq} \ A_{27a} \ A_{27b}) \ V0s) \ V1t)) \wedge (p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardleq} \ A_{27b} \ A_{27c}) \ V1t) \ V2u))) \Rightarrow (p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardleq} \ A_{27a} \ A_{27c}) \ V0s) \ V2u)))))) \Rightarrow (32)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in (2^{A_{27b}}).(((p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardeq} \ A_{27a} \ A_{27b}) \ V0s) \ V1t)) \Leftrightarrow (p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardeq} \ A_{27b} \ A_{27a}) \ V1t) \ V0s)))))) \Rightarrow (33)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in (2^{A_{27b}}).(((p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardeq} \ A_{27a} \ A_{27b}) \ V0s) \ V1t)) \Rightarrow (p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardleq} \ A_{27a} \ A_{27b}) \ V0s) \ V1t)))))) \Rightarrow (34)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow \forall A_{27b}.nonempty \ A_{27b} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in (2^{A_{27b}}).(((p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardleq} \ A_{27a} \ A_{27b}) \ V0s) \ V1t)) \wedge (p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardleq} \ A_{27b} \ A_{27a}) \ V1t) \ V0s))) \Leftrightarrow (p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardeq} \ A_{27a} \ A_{27b}) \ V0s) \ V1t)))))) \Rightarrow (35)$$

Assume the following.

$$\forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}).(\forall V1t \in (2^{A_{27a}}).(((p \ (ap \ (ap \ (c_{2Epred_set_2ESUBSET} \ A_{27a}) \ V0s) \ V1t)) \Rightarrow (p \ (ap \ (ap \ (c_{2Ecardinal_2Ecardleq} \ A_{27a} \ A_{27a}) \ V0s) \ V1t)))))) \Rightarrow (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (p (ap (ap (c_2Ecardinal_2Ecardleq\ A_27a\ A_27a)\ V0s)\ (c_2Epred_set_2EUNIV\ A_27a)))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((\exists V1x \in A_27a. (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V1x)\ V0s))) \Leftrightarrow (\neg (V0s = (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((p (ap\ c_2Ereal_topology_2EOpen\ V0s)) \Leftrightarrow (\forall V1x \in ty_2Erealax_2Ereal. ((p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V1x)\ V0s)) \Rightarrow (\exists V2a \in ty_2Erealax_2Ereal. \\ & (\exists V3b \in ty_2Erealax_2Ereal. ((p (ap (ap (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V1x)\ (ap\ c_2Ereal_topology_2EOPEN_interval\ (ap (ap (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b)))))) \wedge (p (ap (ap (c_2Epred_set_2ESUBSET\ ty_2Erealax_2Ereal)\ (ap\ c_2Ereal_topology_2ECLOSED_interval\ (ap (ap (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal))\ (ap (ap (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b))\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal))))))\ V0s)))))))))) \quad (40) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0a \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Erealax_2Ereal. \\
& ((\neg((ap\ c_2Ereal_topology_2EOPEN_interval\ (ap\ (ap\ (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V0a)\ V1b)) = (c_2Epred_set_2EEMPTY \\
& ty_2Erealax_2Ereal)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ (ap\ c_2Ereal_topology_2ECLOSED_interval \\
& (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal \\
& ty_2Erealax_2Ereal)\ V0a)\ V1b))\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod \\
& ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal))))))\ (c_2Epred_set_2EUNIV \\
& ty_2Erealax_2Ereal)))))) \wedge (\forall V2a \in ty_2Erealax_2Ereal. \\
& (\forall V3b \in ty_2Erealax_2Ereal. ((\neg((ap\ c_2Ereal_topology_2EOPEN_interval \\
& (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& V2a)\ V3b)) = (c_2Epred_set_2EEMPTY\ ty_2Erealax_2Ereal)))) \Rightarrow (\\
& p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& (ap\ c_2Ereal_topology_2EOPEN_interval\ (ap\ (ap\ (c_2Epair_2E_2C \\
& ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ V2a)\ V3b)))\ (c_2Epred_set_2EUNIV \\
& ty_2Erealax_2Ereal)))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False) \Rightarrow False)) \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(\\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee \neg(p \ V2r))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p))))))
\end{aligned} \tag{51}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q)) \Rightarrow (p \ V0p)))) \tag{52}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q)) \Rightarrow \neg(p \ V1q)))) \tag{53}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (((p \ (ap \ c_2Ereal_topology_2EOpen \\
& V0s)) \wedge (\neg(V0s = (c_2Epred_set_2EEMPTY \ ty_2Erealax_2Ereal)))) \Rightarrow \\
& (p \ (ap \ (ap \ (c_2Ecardinal_2Ecard \ ty_2Erealax_2Ereal \ ty_2Erealax_2Ereal) \\
& V0s) \ (c_2Epred_set_2EUNIV \ ty_2Erealax_2Ereal))))))
\end{aligned}$$