

thm_2Ereal__topology_2ECLOSED__FORALL__IN (TMWBa1GpKvUvdtV5giRPF9QnN7Ajjeeqsii)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_2E3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Definition 9 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.a.c_2Ebool_2E_2ET)$.

Definition 10 We define $c_2Ebool_2E_2EIN$ to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x))$

Definition 11 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (3)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (4)$$

Definition 13 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2E$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (7)$$

Definition 14 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap\ (c_2Emin_2E40\ ($

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Definition 15 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP).$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (12)$$

Definition 17 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E2$

Definition 18 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_2Ereal_topo$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (13)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Definition 19 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Definition 20 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_s$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (22)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in A_27b.(((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN A_27a) V2x) V1t)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in ((ty_2Epair_2Eprod A_27a 2)^{A_27b}).(\forall V1v \in A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1v) (ap (c_2Epred_set_2EGSPEC A_27a A_27b) V0f))) \Leftrightarrow (\exists V2x \in A_27b.((ap (ap (c_2Epair_2E_2C A_27a 2) V1v) c_2Ebool_2ET) = (ap V0f V2x)))))) \quad (27)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(((p V0P) \Leftrightarrow (p V1Q)) \Rightarrow ((p V0P) \Rightarrow (p V1Q)))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& (\forall V0Q \in (2^{A.27b}).((\forall V1P \in (2^{A.27a}).(\forall V2f \in \\
& (A.27b^{A.27a}).((\forall V3z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27b)\ V3z)\ (ap\ (c.2Epred_set.2EGSPEC\ A.27b\ A.27a)\ (\lambda V4x \in \\
& A.27a.(ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ 2)\ (ap\ V2f\ V4x))\ (ap\ V1P\ V4x)))))) \Rightarrow \\
& (p\ (ap\ V0Q\ V3z)))) \Leftrightarrow (\forall V5x \in A.27a.((p\ (ap\ V1P\ V5x)) \Rightarrow (p\ (ap\ V0Q \\
& (ap\ V2f\ V5x)))))) \wedge ((\forall V6P \in ((2^{A.27d})^{A.27c}).(\forall V7f \in \\
& ((A.27b^{A.27d})^{A.27c}).((\forall V8z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& A.27b)\ V8z)\ (ap\ (c.2Epred_set.2EGSPEC\ A.27b\ (ty.2Epair.2Eprod \\
& A.27c\ A.27d))\ (ap\ (c.2Epair.2EUNCURRY\ A.27c\ A.27d\ (ty.2Epair.2Eprod \\
& A.27b\ 2))\ (\lambda V9x \in A.27c.(\lambda V10y \in A.27d.(ap\ (ap\ (c.2Epair.2E.2C \\
& A.27b\ 2)\ (ap\ (ap\ V7f\ V9x)\ V10y))\ (ap\ (ap\ V6P\ V9x)\ V10y)))))) \Rightarrow (p \\
& (ap\ V0Q\ V8z)))) \Leftrightarrow (\forall V11x \in A.27c.(\forall V12y \in A.27d.((p \\
& (ap\ (ap\ V6P\ V11x)\ V12y)) \Rightarrow (p\ (ap\ V0Q\ (ap\ (ap\ V7f\ V11x)\ V12y)))))) \wedge \\
& (\forall V13P \in (((2^{A.27g})^{A.27f})^{A.27e}).(\forall V14f \in (((A.27b^{A.27g})^{A.27f})^{A.27e}). \\
& (\forall V15z \in A.27b.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V15z)\ (\\
& ap\ (c.2Epred_set.2EGSPEC\ A.27b\ (ty.2Epair.2Eprod\ A.27e\ (ty.2Epair.2Eprod \\
& A.27f\ A.27g)))\ (ap\ (c.2Epair.2EUNCURRY\ A.27e\ (ty.2Epair.2Eprod \\
& A.27f\ A.27g)\ (ty.2Epair.2Eprod\ A.27b\ 2))\ (\lambda V16w \in A.27e.(ap \\
& (c.2Epair.2EUNCURRY\ A.27f\ A.27g\ (ty.2Epair.2Eprod\ A.27b\ 2)) \\
& (\lambda V17x \in A.27f.(\lambda V18y \in A.27g.(ap\ (ap\ (c.2Epair.2E.2C\ A.27b \\
& 2)\ (ap\ (ap\ (ap\ V14f\ V16w)\ V17x)\ V18y))\ (ap\ (ap\ (ap\ V13P\ V16w)\ V17x) \\
& V18y)))))) \Rightarrow (p\ (ap\ V0Q\ V15z)))) \Leftrightarrow (\forall V19w \in A.27e.(\forall V20x \in \\
& A.27f.(\forall V21y \in A.27g.((p\ (ap\ (ap\ (ap\ V13P\ V19w)\ V20x)\ V21y)) \Rightarrow \\
& (p\ (ap\ V0Q\ (ap\ (ap\ (ap\ V14f\ V19w)\ V20x)\ V21y)))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (2^{(2^{ty.2Erealax.2Ereal})}).((\forall V1s \in (2^{ty.2Erealax.2Ereal}). \\
& ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{ty.2Erealax.2Ereal})\ V1s)\ V0f)) \Rightarrow \\
& (p\ (ap\ c.2Ereal_topology.2EClosed\ V1s)))) \Rightarrow (p\ (ap\ c.2Ereal_topology.2EClosed \\
& (ap\ (c.2Epred_set.2EBIGINTER\ ty.2Erealax.2Ereal)\ V0f))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow \forall A_27e.nonempty \\
& A_27e \Rightarrow \forall A_27f.nonempty\ A_27f \Rightarrow \forall A_27g.nonempty\ A_27g \Rightarrow \\
& \forall A_27h.nonempty\ A_27h \Rightarrow \forall A_27i.nonempty\ A_27i \Rightarrow (\\
& (\forall V0P \in (2^{A_27a}).(\forall V1f \in ((2^{A_27b})^{A_27a}).((ap \\
& (c_2Epred_set_2EBIGINTER\ A_27b)\ (ap\ (c_2Epred_set_2EGSPEC \\
& (2^{A_27b})\ A_27a)\ (\lambda V2x \in A_27a.(ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27b}) \\
& 2)\ (ap\ V1f\ V2x))\ (ap\ V0P\ V2x)))))) = (ap\ (c_2Epred_set_2EGSPEC\ A_27b \\
& A_27b)\ (\lambda V3a \in A_27b.(ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ 2)\ V3a)\ (\\
& ap\ (c_2Ebool_2E_21\ A_27a)\ (\lambda V4x \in A_27a.(ap\ (ap\ c_2Emin_2E_3D_3D_3E \\
& (ap\ V0P\ V4x))\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V3a)\ (ap\ V1f\ V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A_27d})^{A_27c}).(\forall V6f \in (((2^{A_27e})^{A_27d})^{A_27c}). \\
& ((ap\ (c_2Epred_set_2EBIGINTER\ A_27e)\ (ap\ (c_2Epred_set_2EGSPEC \\
& (2^{A_27e})\ (ty_2Epair_2Eprod\ A_27c\ A_27d))\ (ap\ (c_2Epair_2EUNCURRY \\
& A_27c\ A_27d\ (ty_2Epair_2Eprod\ (2^{A_27e})\ 2))\ (\lambda V7x \in A_27c. \\
& (\lambda V8y \in A_27d.(ap\ (ap\ (c_2Epair_2E_2C\ (2^{A_27e})\ 2)\ (ap\ (ap\ V6f \\
& V7x)\ V8y))\ (ap\ (ap\ V5P\ V7x)\ V8y)))))) = (ap\ (c_2Epred_set_2EGSPEC \\
& A_27e\ A_27e)\ (\lambda V9a \in A_27e.(ap\ (ap\ (c_2Epair_2E_2C\ A_27e\ 2) \\
& V9a)\ (ap\ (c_2Ebool_2E_21\ A_27c)\ (\lambda V10x \in A_27c.(ap\ (c_2Ebool_2E_21 \\
& A_27d)\ (\lambda V11y \in A_27d.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ (ap\ (ap\ V5P \\
& V10x)\ V11y))\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27e)\ V9a)\ (ap\ (ap\ V6f\ V10x) \\
& V11y)))))))))) \wedge (\forall V12P \in (((2^{A_27h})^{A_27g})^{A_27f}). \\
& (\forall V13f \in (((((2^{A_27i})^{A_27h})^{A_27g})^{A_27f}).((ap\ (c_2Epred_set_2EBIGINTER \\
& A_27i)\ (ap\ (c_2Epred_set_2EGSPEC\ (2^{A_27i})\ (ty_2Epair_2Eprod \\
& A_27f\ (ty_2Epair_2Eprod\ A_27g\ A_27h)))\ (ap\ (c_2Epair_2EUNCURRY \\
& A_27f\ (ty_2Epair_2Eprod\ A_27g\ A_27h)\ (ty_2Epair_2Eprod\ (2^{A_27i}) \\
& 2))\ (\lambda V14x \in A_27f.(ap\ (c_2Epair_2EUNCURRY\ A_27g\ A_27h\ (ty_2Epair_2Eprod \\
& (2^{A_27i})\ 2))\ (\lambda V15y \in A_27g.(\lambda V16z \in A_27h.(ap\ (ap\ (c_2Epair_2E_2C \\
& (2^{A_27i})\ 2)\ (ap\ (ap\ (ap\ V13f\ V14x)\ V15y)\ V16z))\ (ap\ (ap\ (ap\ V12P\ V14x) \\
& V15y)\ V16z)))))))))) = (ap\ (c_2Epred_set_2EGSPEC\ A_27i\ A_27i) \\
& (\lambda V17a \in A_27i.(ap\ (ap\ (c_2Epair_2E_2C\ A_27i\ 2)\ V17a)\ (ap\ (c_2Ebool_2E_21 \\
& A_27f)\ (\lambda V18x \in A_27f.(ap\ (c_2Ebool_2E_21\ A_27g)\ (\lambda V19y \in \\
& A_27g.(ap\ (c_2Ebool_2E_21\ A_27h)\ (\lambda V20z \in A_27h.(ap\ (ap\ c_2Emin_2E_3D_3D_3E \\
& (ap\ (ap\ (ap\ V12P\ V18x)\ V19y)\ V20z))\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27i) \\
& V17a)\ (ap\ (ap\ (ap\ V13f\ V18x)\ V19y)\ V20z)))))))))))))) \\
& \tag{31}
\end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}). (\forall V1Q \in \\ & \quad ((2^{\text{ty_Erealax_Ereal}})^{A_{27a}}). ((\forall V2a \in A_{27a}. ((p (ap \\ V0P V2a)) \Rightarrow (p (ap \text{c_Ereal_topology_EClosed } (ap (c_Epred_set_EGSPEC \\ \text{ty_Erealax_Ereal ty_Erealax_Ereal } (\lambda V3x \in \text{ty_Erealax_Ereal}. \\ (ap (ap (c_Epair_E_2C \text{ ty_Erealax_Ereal } 2) V3x) (ap (ap V1Q \\ V2a) V3x)))))))))) \Rightarrow (p (ap \text{c_Ereal_topology_EClosed } (ap (c_Epred_set_EGSPEC \\ \text{ty_Erealax_Ereal ty_Erealax_Ereal } (\lambda V4x \in \text{ty_Erealax_Ereal}. \\ (ap (ap (c_Epair_E_2C \text{ ty_Erealax_Ereal } 2) V4x) (ap (c_Ebool_E_21 \\ A_{27a}) (\lambda V5a \in A_{27a}. (ap (ap \text{c_Emin_E_3D_3D_3E } (ap V0P V5a)) \\ (ap (ap V1Q V5a) V4x)))))))))))))) \end{aligned}$$