

thm_2Ereal_topology_2ECLOSED_IN
(TMG9b3XW5zffAdN5EvK6bWj4qvWDWFLt54D)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_2E21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Definition 7 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2ET)$.

Definition 8 We define $c_2Ebool_2E_2EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 10 We define $c_2Epair_2E_2E2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(4)

Definition 11 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27b)\ V0s\ V1t)$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(7)

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ap\ P\ x))\ V0a)$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
(8)

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ap\ P\ x))\ V0T1\ V1T2)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(11)

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
(12)

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ (ap\ P\ x))\ V0s\ V1t))\ V0P))$

Definition 17 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E2$

Definition 18 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_2Ereal_topo$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (13)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

Definition 19 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap (c_2Etopology_2Etopology ty_2Erealax$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Eopen_in A_27a \in ((2^{(2^A-27a)})(ty_2Etopology_2Etopology A_27a)) \quad (15)$$

Definition 20 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^A-27a)}).(ap (c_2Epred_s$

Definition 21 We define $c_2Etopology_2Etopspace$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 22 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap ($

Definition 23 We define $c_2Etopology_2Eclosed_in$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V0s)\ (c.2Epred_set.2EUNIV\ A.27a)))) \quad (21)$$

Assume the following.

$$(\forall V0s \in (2^{ty.2Erealax.2Ereal}).((p\ (ap\ c.2Ereal_topology.2EOpen\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Etopology.2Eopen_in\ ty.2Erealax.2Ereal)\ c.2Ereal_topology.2Eeuclidean)\ V0s)))) \quad (22)$$

Assume the following.

$$((ap\ (c.2Etopology.2Etopspace\ ty.2Erealax.2Ereal)\ c.2Ereal_topology.2Eeuclidean) = (c.2Epred_set.2EUNIV\ ty.2Erealax.2Ereal)) \quad (23)$$

Theorem 1

$$(\forall V0s \in (2^{ty.2Erealax.2Ereal}).((p\ (ap\ c.2Ereal_topology.2EClosed\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Etopology.2Eclosed_in\ ty.2Erealax.2Ereal)\ c.2Ereal_topology.2Eeuclidean)\ V0s))))$$