

thm_2Ereal__topology_2ECLOSED__MAP__FROM__COMPOSITIO (TMSkPhK6abp8bACjnQY4SQjpJ1XQkPk6mqf)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P) V0P))$

Definition 6 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27b}).(ap (ap (c_2Emin_2E_40 A_27a (ap V0f V1g))) V1g)$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (2)$$

Definition 7 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (ap V1t2 V0t1))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (3)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(4)

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal$$
(5)

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)})$$
(6)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(7)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(8)

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})$$
(9)

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(10)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(11)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(12)

Definition 13 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
(13)

Definition 14 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E_2$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

Definition 15 We define $c_2Ereal_topology_2Eeuclidean$ to be $(ap\ (c_2Etopology_2Etopology\ ty_2Erealax\ .$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Eopen_in\ A_27a \in ((2^{(2^A-27a)})^{(ty_2Etopology_2Etopology\ A_27a)}) \quad (15)$$

Definition 16 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2E$

Definition 17 We define $c_2Ereal_topology_2Esubtopology$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 18 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^A-27a)}).(ap\ (c_2Epred_s$

Definition 19 We define $c_2Etopology_2Etopspace$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 20 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 21 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E2F$

Definition 22 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c_2E$

Definition 23 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ ($

Definition 24 We define $c_2Etopology_2Eclosed_in$ to be $\lambda A_27a : \iota.\lambda V0top \in (ty_2Etopology_2Etopology$

Definition 25 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A-27a}).\lambda V1s \in$

Definition 26 We define $c_2Ereal_topology_2Econtinuous_on$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Ereal$

Definition 27 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A_27a.(p (ap V1Q V3x))))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (28) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \\ & \hspace{15em} (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\ & \hspace{15em} (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b)^{A_27a}). \\ & \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \\ & \hspace{15em} (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c)^{A_27b}). (\forall V1g \in (A_27b)^{A_27a}). \\ & \quad (\forall V2s \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a \\ & \quad A_27c)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27c\ A_27b)\ V0f)\ V1g))\ V2s) = \\ & \quad (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & \quad A_27a\ A_27b)\ V1g)\ V2s)))))) \\ & \hspace{15em} (32) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ & \quad A_27a). (\forall V1s \in (2^{A_27a}). (\forall V2t \in (2^{A_27a}). ((p\ (\\ & \quad ap\ (ap\ (c_2Etopology_2Eclosed_in\ A_27a)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology \\ & \quad A_27a)\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a) \\ & \quad V2t)\ V1s)))))) \\ & \hspace{15em} (33) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}).(\forall V2t \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V3u \in (2^{ty_2Erealax_2Ereal}).(((p (ap (ap (c_2Ereal_topology_2Econtinuous_on \\
& \quad V0f) V1s)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad V0f) V1s)) V2t)) \wedge (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\
& \quad (ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean) V2t)) V3u)))))) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in \\
& \quad ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V1s)) \\
& \quad (ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& \quad (\lambda V4x \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
& \quad 2) V4x) (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
& \quad V4x) V1s)) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V4x)) \\
& \quad V3u)))))))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\
& \quad (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
& \quad p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& \quad ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee \neg(p \vee V2r))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee \neg(p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (42)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge (((p \vee V0p) \vee \neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (43)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow \neg(p \vee V1q)) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge (\neg(p \vee V1q) \vee \neg(p \vee V0p)))))) \\
& \hspace{15em} (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p)))) \\
& \hspace{15em} (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow \neg(p \vee V1q)))) \\
& \hspace{15em} (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V0p)))) \\
& \hspace{15em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V1q)))) \\
& \hspace{15em} (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \\
& \hspace{15em} (49)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})). (\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V2s \in (2^{ty_2Erealax_2Ereal}). \\
& (\forall V3t \in (2^{ty_2Erealax_2Ereal}). (\forall V4u \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) (ap \\
& (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
V0f) V2s)) V3t)) \wedge ((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
V1g) V3t)) V4u)) \wedge ((p (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& V1g) V3t)) \wedge ((\forall V5x \in ty_2Erealax_2Ereal. (\forall V6y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V5x) V3t)) \wedge ((\\
& p (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) V6y) V3t)) \wedge ((ap V1g \\
V5x) = (ap V1g V6y)))) \Rightarrow (V5x = V6y)))) \wedge (\forall V7k \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) (\\
& ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
c_2Ereal_topology_2Eeuclidean) V2s)) V7k)) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in \\
& ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V4u)) \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Ecombin_2Eo ty_2Erealax_2Ereal ty_2Erealax_2Ereal \\
ty_2Erealax_2Ereal) V1g) V0f)) V7k)))))) \Rightarrow (\forall V8k \in (2^{ty_2Erealax_2Ereal}). \\
& ((p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) (\\
& ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
c_2Ereal_topology_2Eeuclidean) V2s)) V8k)) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in \\
& ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
& ty_2Erealax_2Ereal) c_2Ereal_topology_2Eeuclidean) V3t)) \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0f) V8k)))))))))
\end{aligned}$$