

# thm\_2Ereal\_\_topology\_2ECLOSED\_\_MAP\_\_FROM\_\_COMPOSITIC (TMcY6aRRtd3nbPk8eKMQ5YM7vomTyNa3tN8)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}) P) P))))$

**Definition 6** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27c^{A\_27a}).(ap V1g (ap V0f (ap (c\_2Ecombin\_2Eo A\_27a A\_27b) A\_27c)))$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (2)$$

**Definition 7** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (3)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
 (4)

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty ty\_2Erealx\_2Ereal$$
 (5)

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod ty\_2Erealx\_2Ereal ty\_2Erealx\_2Ereal)})$$
 (6)

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty ty\_2Ehreal\_2Ehreal$$
 (7)

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal})$$
 (8)

**Definition 11** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal. (ap (c\_2Emin\_2E\_40 (t$

Let  $c\_2Erealx\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)})$$
 (9)

**Definition 12** We define  $c\_2Erealx\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal. \lambda V1T2 \in ty\_2Erealx\_2Ereal.$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega$$
 (10)

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum$$
 (11)

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega})$$
 (12)

**Definition 13** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP).$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})$$
 (13)

**Definition 14** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealx\_2Ereal}). (ap (c\_2Ebool\_2E\_2$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

**Definition 15** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax\ .$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in\ A\_27a \in ((2^{(2^A-27a)})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \quad (15)$$

**Definition 16** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_2E$

**Definition 17** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 18** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^A-27a)}).(ap\ (c\_2Epred\_s$

**Definition 19** We define  $c\_2Etopology\_2Etopspace$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 20** We define  $c\_2Ebool\_2E2F$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 21** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E2F$

**Definition 22** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ (c\_2E$

**Definition 23** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap\ ($

**Definition 24** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A\_27a : \iota.\lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 25** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A-27a}).\lambda V1s \in$

**Definition 26** We define  $c\_2Ereal\_topology\_2Econtinuous\_on$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Ereal$

**Definition 27** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(V0x = V0x)) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A.27a}).(((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V1P V3x))) \vee (p V0Q)))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((\forall V2x \in A.27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (28)$$

Assume the following.

$$2.((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (29)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (30)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap (ap (c_{.2Epair}_{.2E_{.2C}} A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap (c_{.2Epair}_{.2E_{.2C}} A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (31)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2Ebool}_{.2EIN} A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2Ebool}_{.2EIN} A_{.27a}) V2x) V1t)))))) \quad (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2Epair}_{.2Eprod} A_{.27a} 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p (ap (ap (c_{.2Ebool}_{.2EIN} A_{.27a}) V1v) (ap (c_{.2Epred}_{.set}_{.2EGSPEC} A_{.27a} A_{.27b}) V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap (ap (c_{.2Epair}_{.2E_{.2C}} A_{.27a} 2) V1v) c_{.2Ebool}_{.2ET}) = (ap V0f V2x)))))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p (ap (ap (c_{.2Epred}_{.set}_{.2ESUBSET} A_{.27a}) V0s) V0s))) \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0y \in A_{.27b}.(\forall V1s \in (2^{A_{.27a}}).(\forall V2f \in (A_{.27b}^{A_{.27a}}).((p (ap (ap (c_{.2Ebool}_{.2EIN} A_{.27b}) V0y) (ap (ap (c_{.2Epred}_{.set}_{.2EIMAGE} A_{.27a} A_{.27b}) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_{.27a}.((V0y = (ap V2f V3x)) \wedge (p (ap (ap (c_{.2Ebool}_{.2EIN} A_{.27a}) V3x) V1s)))))) \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27b}). (\forall V1g \in (A\_27b^{A\_27a}). \\
& (\forall V2s \in (2^{A\_27a}). ((ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27a \\
& A\_27c)\ (ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27c\ A\_27b)\ V0f)\ V1g))\ V2s) = \\
& (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& A\_27a\ A\_27b)\ V1g)\ V2s))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\
& A\_27a). (\forall V1s \in (2^{A\_27a}). (\forall V2t \in (2^{A\_27a}). ((p\ ( \\
& ap\ (ap\ (c\_2Etopology\_2Eclosed\_in\ A\_27a)\ (ap\ (ap\ (c\_2Ereal\_topology\_2Esubtopology \\
& A\_27a)\ V0top)\ V1s))\ V2t)) \Rightarrow (p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a) \\
& V2t)\ V1s))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Erealty\_2Erealax\_2Ereal). (\forall V1s \in \\
& (2^{ty\_2Erealax\_2Ereal}). (\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& (\forall V3u \in (2^{ty\_2Erealax\_2Ereal}). (((p\ (ap\ (ap\ c\_2Ereal\_topology\_2Econtinuous\_on \\
& V0f)\ V1s)) \wedge ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ ty\_2Erealax\_2Ereal) \\
& (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal) \\
& V0f)\ V1s))\ V2t)) \wedge (p\ (ap\ (ap\ (c\_2Etopology\_2Eclosed\_in\ ty\_2Erealax\_2Ereal) \\
& (ap\ (ap\ (c\_2Ereal\_topology\_2Esubtopology\ ty\_2Erealax\_2Ereal) \\
& c\_2Ereal\_topology\_2Eeuclidean)\ V2t))\ V3u)))))) \Rightarrow (p\ (ap\ (ap\ (c\_2Etopology\_2Eclosed\_in \\
& ty\_2Erealax\_2Ereal)\ (ap\ (ap\ (c\_2Ereal\_topology\_2Esubtopology \\
& ty\_2Erealax\_2Ereal)\ c\_2Ereal\_topology\_2Eeuclidean)\ V1s)) \\
& (ap\ (c\_2Epred\_set\_2EGSPEC\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal) \\
& (\lambda V4x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Erealax\_2Ereal \\
& 2)\ V4x)\ (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ty\_2Erealax\_2Ereal) \\
& V4x)\ V1s))\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ty\_2Erealax\_2Ereal)\ (ap\ V0f\ V4x)) \\
& V3u))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (53)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\ & (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2s \in (2^{ty\_2Erealax\_2Ereal}). \\ & (\forall V3t \in (2^{ty\_2Erealax\_2Ereal}).(\forall V4u \in (2^{ty\_2Erealax\_2Ereal}). \\ & (((p (ap (ap (c\_2Ereal\_topology\_2Econtinuous\_on V0f) V2s)) \wedge \\ & (((ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & V0f) V2s) = V3t) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\ & (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & V1g) V3t)) V4u)) \wedge (\forall V5k \in (2^{ty\_2Erealax\_2Ereal}).((p (ap \\ & (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) (ap (ap ( \\ & c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) \\ & V2s)) V5k)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) \\ & (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\ & c\_2Ereal\_topology\_2Euclidean) V4u)) (ap (ap (c\_2Epred\_set\_2EIMAGE \\ & ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ecombin\_2Eo \\ & ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & V1g) V0f)) V5k)))))) \Rightarrow (\forall V6k \in (2^{ty\_2Erealax\_2Ereal}). \\ & ((p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) ( \\ & ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\ & c\_2Ereal\_topology\_2Euclidean) V3t)) V6k)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eclosed\_in \\ & ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\ & ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) V4u)) \\ & (ap (ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & V1g) V6k))))))))) \end{aligned}$$