

thm_2Ereal__topology_2ECLOSED__MAP__IMP__QUOTIENT__MA
 (TMJQfHeKR-
 WvP4uhRnEAwJvUonTVZBscLPFJ)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))))$

Definition 5 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 6 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}) P))))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(3)

Definition 11 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$ Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealx_2Ereal$$
(4)

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod ty_2Erealx_2Ereal ty_2Erealx_2Ereal)})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal})$$
(7)

Definition 12 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap (c_2Emin_2E_40 ($

Let $c_2Erealx_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etreallt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})$$
(8)

Definition 13 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(11)

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum})$$
(12)

Definition 15 We define $c_Ereal_topology_EOpen$ to be $\lambda V0s \in (2^{ty_Erealax_Ereal}).(ap (c_Ebool_E2$

Let $ty_Etopology_Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_Etopology_Etopology A0) \quad (13)$$

Let $c_Etopology_Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Etopology_Etopology A_27a \in ((ty_Etopology_Etopology A_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

Definition 16 We define $c_Ereal_topology_Eeuclidean$ to be $(ap (c_Etopology_Etopology ty_Erealax$

Let $c_Etopology_Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Etopology_Eopen_in A_27a \in ((2^{(2^A-27a)})^{(ty_Etopology_Etopology A_27a)}) \quad (15)$$

Definition 17 We define $c_Epred_set_EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c$

Definition 18 We define $c_Ereal_topology_Esubtopology$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_Etopology_Etopology$

Definition 19 We define $c_Ereal_topology_Econtinuous_on$ to be $\lambda V0f \in (ty_Erealax_Ereal)^{ty_Erealax$

Definition 20 We define c_Ebool_E2E to be $(ap (c_Ebool_E2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 21 We define $c_Ebool_E2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_E2E_21 2) (\lambda V2t \in 2. ($

Definition 22 We define $c_Ebool_E2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_E2E$

Definition 23 We define $c_Epred_set_EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^A-27a)}). (ap (c_Epred_set$

Definition 24 We define $c_Etopology_Etopspace$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_Etopology_Etopology$

Definition 25 We define $c_Epred_set_ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap ($

Definition 26 We define $c_Epred_set_EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c_2$

Definition 27 We define $c_Etopology_Eclosed_in$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_Etopology_Etopology$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))) \wedge (((\neg(p \ V0A)) \vee (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \wedge (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V1B) \wedge (p \ V2C)) \vee (p \ V0A) \Leftrightarrow (((p \ V1B) \vee (p \ V0A)) \wedge ((p \ V2C) \vee (p \ V0A)))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (29)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))) \quad (30)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (31)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap (ap (c_{.2E}pair_{.2E} C A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap (c_{.2E}pair_{.2E} C A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V2x) V0s)) \Leftrightarrow (p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V2x) V1t))))))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2E}pair_{.2E} prod A_{.27a} 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V1v) (ap (c_{.2E}pred_{.set} 2EGSPEC A_{.27a} A_{.27b}) V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap (ap (c_{.2E}pair_{.2E} C A_{.27a} 2) V1v) c_{.2E}bool_{.2E} ET) = (ap V0f V2x)))))) \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((p (ap (ap (c_{.2E}pred_{.set} 2ESUBSET A_{.27a}) V0s) (c_{.2E}pred_{.set} 2EUNIV A_{.27a})))))) \quad (35)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V2x) (ap (ap (c_{.2E}pred_{.set} 2EDIFF A_{.27a}) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V2x) V0s)) \wedge (\neg (p (ap (ap (c_{.2E}bool_{.2E} IN A_{.27a}) V2x) V1t)))))))))) \quad (36)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0P \in 2. (\forall V1Q \in 2. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \Rightarrow ((p\ V0P) \Rightarrow (p\ V1Q)))))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\
& \quad A_27a). (\forall V1u \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Etopology_2Eclosed_in \\
& \quad A_27a)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology\ A_27a)\ V0top) \\
& \quad V1u)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V1u)\ (ap \\
& \quad (c_2Etopology_2Etopspace\ A_27a)\ V0top))))))
\end{aligned} \tag{39}$$

Assume the following.

$$((ap\ (c_2Etopology_2Etopspace\ ty_2Erealax_2Ereal)\ c_2Ereal_topology_2Eeuclidean) = (c_2Epred_set_2EUNIV\ ty_2Erealax_2Ereal)) \tag{40}$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((ap\ (c_2Etopology_2Etopspace\ ty_2Erealax_2Ereal)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology\ ty_2Erealax_2Ereal)\ c_2Ereal_topology_2Eeuclidean)\ V0s)) = V0s)) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1s \in \\
& \quad (2^{ty_2Erealax_2Ereal}). ((p\ (ap\ (ap\ c_2Ereal_topology_2Econtinuous_on \\
& \quad V0f)\ V1s)) \Leftrightarrow (\forall V2t \in (2^{ty_2Erealax_2Ereal}). ((p\ (ap\ (ap\ (\\
& \quad c_2Etopology_2Eopen_in\ ty_2Erealax_2Ereal)\ (ap\ (ap\ (c_2Ereal_topology_2Esubtopology \\
& \quad ty_2Erealax_2Ereal)\ c_2Ereal_topology_2Eeuclidean)\ (ap\ (ap \\
& \quad (c_2Epred_set_2EIMAGE\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
& \quad V0f)\ V1s))) \Rightarrow (p\ (ap\ (ap\ (c_2Etopology_2Eopen_in\ ty_2Erealax_2Ereal) \\
& \quad (ap\ (ap\ (c_2Ereal_topology_2Esubtopology\ ty_2Erealax_2Ereal) \\
& \quad c_2Ereal_topology_2Eeuclidean)\ V1s))\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)\ (\lambda V3x \in ty_2Erealax_2Ereal. \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ 2)\ V3x)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ V3x)\ V1s))\ (ap\ (ap\ (\\
& \quad c_2Ebool_2EIN\ ty_2Erealax_2Ereal)\ (ap\ V0f\ V3x))\ V2t))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty \ A_{27a} \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\ A_{27a}).(\forall V1s \in (2^{A_{27a}}).(\forall V2t \in (2^{A_{27a}}).(((p \\ (ap \ (ap \ (c_2Etopology_2Eclosed_in \ A_{27a}) \ V0top) \ V1s)) \wedge (p \ (ap \\ (ap \ (c_2Etopology_2Eopen_in \ A_{27a}) \ V0top) \ V2t))) \Rightarrow (p \ (ap \ (ap \ (\\ c_2Etopology_2Eclosed_in \ A_{27a}) \ V0top) \ (ap \ (ap \ (c_2Epred_set_2EDIFF \\ A_{27a}) \ V1s) \ V2t)))))))) \end{aligned} \quad (58)$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).((p (ap (ap (ap c_2Ereal_topology_2Econtinuous_on \\
& V0f) V1s)) \wedge (\forall V2t \in (2^{ty_2Erealax_2Ereal}).((p (ap (ap (\\
c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V1s)) \\
V2t)) \Rightarrow (p (ap (ap (c_2Etopology_2Eclosed_in ty_2Erealax_2Ereal) \\
(ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
c_2Ereal_topology_2Euclidean) (ap (ap (c_2Epred_set_2EIMAGE \\
ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V1s))) (ap (ap (c_2Epred_set_2EIMAGE \\
ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V2t)))))) \Rightarrow (\forall V3t \in \\
(2^{ty_2Erealax_2Ereal}).((p (ap (ap (c_2Epred_set_2ESUBSET \\
ty_2Erealax_2Ereal) V3t) (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal \\
ty_2Erealax_2Ereal) V0f) V1s))) \Rightarrow ((p (ap (ap (c_2Etopology_2Eopen_in \\
ty_2Erealax_2Ereal) (ap (ap (c_2Ereal_topology_2Esubtopology \\
ty_2Erealax_2Ereal) c_2Ereal_topology_2Euclidean) V1s)) \\
(ap (c_2Epred_set_2EGSPEC ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
(\lambda V4x \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\
2) V4x) (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) \\
V4x) V1s)) (ap (ap (c_2Ebool_2EIN ty_2Erealax_2Ereal) (ap V0f V4x)) \\
V3t)))))) \Leftrightarrow (p (ap (ap (c_2Etopology_2Eopen_in ty_2Erealax_2Ereal) \\
(ap (ap (c_2Ereal_topology_2Esubtopology ty_2Erealax_2Ereal) \\
c_2Ereal_topology_2Euclidean) (ap (ap (c_2Epred_set_2EIMAGE \\
ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V1s))) V3t))))))
\end{aligned}$$