

# thm\_2Ereal\_\_topology\_2ECLOSED\_\_MAP\_\_OPEN\_\_SUPERSET\_\_P (TMF7Lcg1XJPUf4dtTNsQ9e3rf4tCShMmU2F)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ . **if**  $(\exists x \in A.p (ap P x))$  **then** (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EC$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A\_27a})).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1f \in (2^{A\_27a}))))$

**Definition 7** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda A.\lambda c : \iota.(\lambda V0f \in (A\_27b^{A\_27c})).\lambda V1g \in (A\_27c^{A\_27a})$

**Definition 8** We define  $c\_2Ebool\_2E\_F$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 9** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A.\lambda a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_F)$ .

**Definition 10** We define  $c\_2Ebool\_2E\_IN$  to be  $\lambda A.\lambda a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a})).(ap V1f V0x))$

**Definition 11** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (1)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (2)$$

**Definition 14** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (3)$$

**Definition 15** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2E$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (4)$$

Let  $c\_2Ereal\_topology\_2EDist : \iota$  be given. Assume the following.

$$c\_2Ereal\_topology\_2EDist \in (ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Erealx\_2Ereal\ ty\_2Erealx\_2Ereal)}) \quad (5)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (6)$$

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal}) \quad (7)$$

**Definition 16** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ ($

Let  $c\_2Erealx\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

**Definition 17** We define  $c\_2Erealx\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.\lambda V1T2 \in ty\_2Erealx\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 18** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 19** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 20** We define  $c\_2Ereal\_topology\_2EOpen$  to be  $\lambda V0s \in (2^{ty\_2Erealax\_2Ereal}). (ap\ (c\_2Ebool\_2E\_2$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (13)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^A-27a)})}) \quad (14)$$

**Definition 21** We define  $c\_2Ereal\_topology\_2Eeuclidean$  to be  $(ap\ (c\_2Etopology\_2Etopology\ ty\_2Erealax\_2Ereal$

Let  $c\_2Etopology\_2Eopen\_in : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Eopen\_in\ A\_27a \in ((2^{(2^A-27a)})^{(ty\_2Etopology\_2Etopology\ A\_27a)}) \quad (15)$$

**Definition 22** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c\_2E$

**Definition 23** We define  $c\_2Ereal\_topology\_2Esubtopology$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 24** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^A-27a)}). (ap\ (c\_2Epred\_set$

**Definition 25** We define  $c\_2Etopology\_2Etopospace$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 26** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E$

**Definition 27** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ (c\_2E$

**Definition 28** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap\ ($

**Definition 29** We define  $c\_2Etopology\_2Eclosed\_in$  to be  $\lambda A\_27a : \iota. \lambda V0top \in (ty\_2Etopology\_2Etopology$

**Definition 30** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A-27a}). \lambda V1s \in$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (16)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (17)$$

**Definition 31** We define  $c\_2\text{Epair\_2EUNCURRY}$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a}$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (19)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (22)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (23)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p V0P) \wedge (\forall V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x))))))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A-27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x)) \vee (p V0Q)))))) \quad (29)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A-27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (33)$$

Assume the following.

$$2. (((p V0x) \Leftrightarrow (p V1x_{-27})) \wedge ((p V1x_{-27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{-27}))) \Rightarrow 2. (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{-27}) \Rightarrow (p V3y_{-27})))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1a \in A.27a. ((\exists V2x \in A.27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in ((2^{A-27b})^{A-27a}). ((\forall V1x \in A.27a. (\exists V2y \in A.27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A-27a}). (\forall V4x \in A.27a. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & 2. (((\exists V2x \in A\_27a. (p\ (ap\ V0P\ V2x))) \Rightarrow (p\ V1Q))) \Leftrightarrow (\forall V3x \in \\ & A\_27a. ((p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1x \in \\ & A\_27a. ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ & A\_27a)\ V1x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ ( \\ & c\_2Ebool\_2EIN\ A\_27a)\ V1x)\ V0s)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\ & A\_27b. (((ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap \\ & (c\_2Epair\_2E\_2C\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}). (\forall V1v \in \\ & A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\ & A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ & A\_27a)\ V1y)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0y \in A\_27b. (\forall V1s \in (2^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}). \\ & ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\ & A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap ( \\
& \quad ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad V0f) V1s)) V2t)) \Rightarrow ((\forall V3k \in (2^{ty\_2Erealax\_2Ereal}).((p ( \\
& \quad ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) (ap (ap \\
& (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) \\
& V1s) V3k)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Euclidean) V2t)) (ap (ap (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0f) V3k)))))) \Leftrightarrow (\forall V4u \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V5w \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad (((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Euclidean) V1s)) V4u)) \wedge ((p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& \quad ty\_2Erealax\_2Ereal) V5w) V2t)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& \quad ty\_2Erealax\_2Ereal) (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) (\lambda V6x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C \\
& \quad ty\_2Erealax\_2Ereal 2) V6x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V6x) V1s)) (ap (ap (c\_2Ebool\_2EIN \\
& \quad ty\_2Erealax\_2Ereal) (ap V0f V6x)) V5w)))))) V4u)))))) \Rightarrow (\exists V7v \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Etopology\_2Eopen\_in \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) V2t)) \\
& \quad V7v)) \wedge ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& \quad V5w) V7v)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) \\
& \quad (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad (\lambda V8x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& \quad 2) V8x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) \\
& \quad V8x) V1s)) (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V8x)) \\
& \quad V7v)))))) V4u)))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\
& A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow \forall A.27g.nonempty\ A.27g \Rightarrow \\
& \forall A.27h.nonempty\ A.27h \Rightarrow \forall A.27i.nonempty\ A.27i \Rightarrow ( \\
& (\forall V0P \in (2^{A.27a}).(\forall V1f \in ((2^{A.27b})^{A.27a}).((ap \\
& (c.2Epred\_set\_2EBIGUNION\ A.27b)\ (ap\ (c.2Epred\_set\_2EGSPEC \\
& (2^{A.27b})\ A.27a)\ (\lambda V2x \in A.27a.(ap\ (ap\ (c.2Epair\_2E\_2C\ (2^{A.27b}) \\
& 2)\ (ap\ V1f\ V2x))\ (ap\ V0P\ V2x)))))) = (ap\ (c.2Epred\_set\_2EGSPEC\ A.27b \\
& A.27b)\ (\lambda V3a \in A.27b.(ap\ (ap\ (c.2Epair\_2E\_2C\ A.27b\ 2)\ V3a)\ ( \\
& ap\ (c.2Ebool\_2E\_3F\ A.27a)\ (\lambda V4x \in A.27a.(ap\ (ap\ c.2Ebool\_2E\_2F\_5C \\
& (ap\ V0P\ V4x))\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27b)\ V3a)\ (ap\ V1f\ V4x)))))))))) \wedge \\
& ((\forall V5P \in ((2^{A.27d})^{A.27c}).(\forall V6f \in (((2^{A.27e})^{A.27d})^{A.27c}). \\
& ((ap\ (c.2Epred\_set\_2EBIGUNION\ A.27e)\ (ap\ (c.2Epred\_set\_2EGSPEC \\
& (2^{A.27e})\ (ty\_2Epair\_2Eprod\ A.27c\ A.27d))\ (ap\ (c.2Epair\_2EUNCURRY \\
& A.27c\ A.27d\ (ty\_2Epair\_2Eprod\ (2^{A.27e})\ 2))\ (\lambda V7x \in A.27c. \\
& (\lambda V8y \in A.27d.(ap\ (ap\ (c.2Epair\_2E\_2C\ (2^{A.27e})\ 2)\ (ap\ (ap\ V6f \\
& V7x)\ V8y))\ (ap\ (ap\ V5P\ V7x)\ V8y)))))) = (ap\ (c.2Epred\_set\_2EGSPEC \\
& A.27e\ A.27e)\ (\lambda V9a \in A.27e.(ap\ (ap\ (c.2Epair\_2E\_2C\ A.27e\ 2) \\
& V9a)\ (ap\ (c.2Ebool\_2E\_3F\ A.27c)\ (\lambda V10x \in A.27c.(ap\ (c.2Ebool\_2E\_3F \\
& A.27d)\ (\lambda V11y \in A.27d.(ap\ (ap\ c.2Ebool\_2E\_2F\_5C\ (ap\ (ap\ V5P\ V10x) \\
& V11y))\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27e)\ V9a)\ (ap\ (ap\ V6f\ V10x)\ V11y)))))))))) \wedge \\
& ((\forall V12P \in (((2^{A.27h})^{A.27g})^{A.27f}).(\forall V13f \in (((2^{A.27i})^{A.27h})^{A.27g})^{A.27f}). \\
& ((ap\ (c.2Epred\_set\_2EBIGUNION\ A.27i)\ (ap\ (c.2Epred\_set\_2EGSPEC \\
& (2^{A.27i})\ (ty\_2Epair\_2Eprod\ A.27f\ (ty\_2Epair\_2Eprod\ A.27g\ A.27h)) \\
& (ap\ (c.2Epair\_2EUNCURRY\ A.27f\ (ty\_2Epair\_2Eprod\ A.27g\ A.27h) \\
& (ty\_2Epair\_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V14x \in A.27f.(ap\ (c.2Epair\_2EUNCURRY \\
& A.27g\ A.27h\ (ty\_2Epair\_2Eprod\ (2^{A.27i})\ 2))\ (\lambda V15y \in A.27g. \\
& (\lambda V16z \in A.27h.(ap\ (ap\ (c.2Epair\_2E\_2C\ (2^{A.27i})\ 2)\ (ap\ (ap \\
& (ap\ V13f\ V14x)\ V15y)\ V16z))\ (ap\ (ap\ (ap\ V12P\ V14x)\ V15y)\ V16z)))))) = \\
& (ap\ (c.2Epred\_set\_2EGSPEC\ A.27i\ A.27i)\ (\lambda V17a \in A.27i.(ap \\
& (ap\ (c.2Epair\_2E\_2C\ A.27i\ 2)\ V17a)\ (ap\ (c.2Ebool\_2E\_3F\ A.27f) \\
& (\lambda V18x \in A.27f.(ap\ (c.2Ebool\_2E\_3F\ A.27g)\ (\lambda V19y \in A.27g. \\
& (ap\ (c.2Ebool\_2E\_3F\ A.27h)\ (\lambda V20z \in A.27h.(ap\ (ap\ c.2Ebool\_2E\_2F\_5C \\
& (ap\ (ap\ (ap\ V12P\ V18x)\ V19y)\ V20z))\ (ap\ (ap\ (c.2Ebool\_2EIN\ A.27i) \\
& V17a)\ (ap\ (ap\ (ap\ V13f\ V18x)\ V19y)\ V20z)))))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{45}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{46}$$



Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0top \in (ty\_2Etopology\_2Etopology \\ & \quad A.27a).(\forall V1k \in (2^{(2^{A.27a})}).(\forall V2s \in (2^{A.27a}). \\ & ((p (ap (ap (c\_2Ebool\_2EIN (2^{A.27a}) V2s) V1k)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eopen\_in \\ & \quad A.27a) V0top) V2s)))) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eopen\_in A.27a) \\ & \quad V0top) (ap (c\_2Epred\_set\_2EBIGUNION A.27a) V1k)))))) \end{aligned} \quad (60)$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1s \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V2t \in (2^{ty\_2Erealax\_2Ereal}). \\
& \quad ((p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap ( \\
& \quad ap (c\_2Epred\_set\_2EIMAGE ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\
& \quad V0f) V1s)) V2t)) \Rightarrow ((\forall V3k \in (2^{ty\_2Erealax\_2Ereal}).((p ( \\
& \quad ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) (ap (ap \\
& (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) \\
& V1s)) V3k)) \Rightarrow (p (ap (ap (c\_2Etopology\_2Eclosed\_in ty\_2Erealax\_2Ereal) \\
& \quad (ap (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Euclidean) V2t)) (ap (ap (c\_2Epred\_set\_2EIMAGE \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0f) V3k)))))) \Leftrightarrow (\forall V4u \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).(\forall V5y \in ty\_2Erealax\_2Ereal.( \\
& \quad ((p (ap (ap (c\_2Etopology\_2Eopen\_in ty\_2Erealax\_2Ereal) (ap \\
& \quad (ap (c\_2Ereal\_topology\_2Esubtopology ty\_2Erealax\_2Ereal) \\
& \quad c\_2Ereal\_topology\_2Euclidean) V1s)) V4u)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN \\
& \quad ty\_2Erealax\_2Ereal) V5y) V2t)) \wedge (p (ap (ap (c\_2Epred\_set\_2ESUBSET \\
& \quad ty\_2Erealax\_2Ereal) (ap (c\_2Epred\_set\_2EGSPEC ty\_2Erealax\_2Ereal \\
& \quad ty\_2Erealax\_2Ereal) (\lambda V6x \in ty\_2Erealax\_2Ereal.(ap (ap (c\_2Epair\_2E\_2C \\
& \quad ty\_2Erealax\_2Ereal 2) V6x) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V6x) V1s)) (ap (ap (c\_2Emin\_2E\_3D \\
& \quad ty\_2Erealax\_2Ereal) (ap V0f V6x)) V5y)))))) V4u)))) \Rightarrow (\exists V7v \in \\
& \quad (2^{ty\_2Erealax\_2Ereal}).((p (ap (ap (c\_2Etopology\_2Eopen\_in \\
& \quad ty\_2Erealax\_2Ereal) (ap (ap (c\_2Ereal\_topology\_2Esubtopology \\
& \quad ty\_2Erealax\_2Ereal) c\_2Ereal\_topology\_2Euclidean) V2t)) \\
& \quad V7v)) \wedge ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V5y) V7v)) \wedge \\
& (p (ap (ap (c\_2Epred\_set\_2ESUBSET ty\_2Erealax\_2Ereal) (ap (c\_2Epred\_set\_2EGSPEC \\
& \quad ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (\lambda V8x \in ty\_2Erealax\_2Ereal. \\
& \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal 2) V8x) (ap (ap c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap (ap (c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) V8x) V1s)) (ap (ap ( \\
& \quad c\_2Ebool\_2EIN ty\_2Erealax\_2Ereal) (ap V0f V8x)) V7v)))))) V4u)))))))))
\end{aligned}$$