

thm_2Ereal_topology_2ECLOSED_UNION (TMVj4ARn2edCh91hHDyaad47CUkduy7uJRn)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Definition 7 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)}) \quad (4)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 11 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (ap\ V0a)))$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Definition 12 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(ap\ (c_2Erealax_2Etreallt\ (ap\ V0T1)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (10)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (11)$$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ (ap\ V0P))))$

Definition 15 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap\ (c_2Ebool_2E_3F\ (ap\ V0s)))$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (12)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Etopology_2Etopology\ A.27a \in ((ty_2Etopology_2Etopology\ A.27a)^{(2^{(2^A-27a)}})) \quad (13)$$

Definition 16 We define $c_2Ereal_topology_2Euclidean$ to be $(ap (c_2Etopology_2Etopology ty_2Erealax$

Definition 17 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (14)$$

Definition 18 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2E$

Definition 19 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap c_2Ereal_topo$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 21 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2E$

Let $c_2Etopology_2Eopen_in : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Eopen_in A_27a \in \\ ((2^{(2^{A_27a})})^{(ty_2Etopology_2Etopology A_27a)}) \quad (15)$$

Definition 22 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap (c_2Epred_set_2E$

Definition 23 We define $c_2Etopology_2Etopspace$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Definition 24 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2E$

Definition 25 We define $c_2Etopology_2EClosed_in$ to be $\lambda A_27a : \iota. \lambda V0top \in (ty_2Etopology_2Etopology$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg (\\ p V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}). ((p (ap c_2Ereal_topology_2EClosed \\ V0s)) \Leftrightarrow (p (ap (ap (c_2Etopology_2EClosed_in ty_2Erealax_2Ereal) \\ c_2Ereal_topology_2Euclidean) V0s)))) \quad (19)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0top \in (ty_2Etopology_2Etopology \\
& \quad A_{27a}). (\forall V1s \in (2^{A_{27a}}). (\forall V2t \in (2^{A_{27a}}). (((p \\
& \quad (ap (ap (c_2Etopology_2EClosed_in A_{27a}) V0top) V1s)) \wedge (p (ap \\
& \quad (ap (c_2Etopology_2EClosed_in A_{27a}) V0top) V2t))) \Rightarrow (p (ap (ap \\
& (c_2Etopology_2EClosed_in A_{27a}) V0top) (ap (ap (c_2Epred_set_2EUNION \\
& \quad A_{27a}) V1s) V2t))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}). (\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\
& (((p (ap c_2Ereal_topology_2EClosed V0s)) \wedge (p (ap c_2Ereal_topology_2EClosed \\
& V1t))) \Rightarrow (p (ap c_2Ereal_topology_2EClosed (ap (ap (c_2Epred_set_2EUNION \\
& \quad ty_2Erealax_2Ereal) V0s) V1t))))))
\end{aligned}$$