

thm_2Ereal__topology_2ECLOSURE__BOUNDED__LINEAR__IMA
(TMY4cc34UWzAY6iGrLw6JGBfS7c8ocshtbu)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Definition 7 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2T)$.

Definition 8 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{3}$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(4)

Definition 11 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(7)

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal. (ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)})$$
(8)

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega$$
(9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega})$$
(11)

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
(12)

Definition 16 We define $c_Ebool_E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_E_40$

Definition 17 We define $c_Ereal_topology_E_Open$ to be $\lambda V0s \in (2^{ty_Erealax_E_Real}). (ap\ (c_Ebool_E_2$

Definition 18 We define $c_Ereal_topology_E_Closed$ to be $\lambda V0s \in (2^{ty_Erealax_E_Real}). (ap\ c_Ereal_topo$

Let $c_Erealax_E_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_E_Etrealm_neg \in ((ty_Epair_E_Eprod\ ty_Ehreal_E_Ehreal\ ty_Ehreal_E_Ehreal) (ty_Epair_E_Eprod\ ty_Ehreal_E_Ehreal\ ty_Ehreal_E_Ehreal)) \quad (13)$$

Let $c_Erealax_E_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_E_Etrealm_eq \in ((2^{(ty_Epair_E_Eprod\ ty_Ehreal_E_Ehreal\ ty_Ehreal_E_Ehreal)}) (ty_Epair_E_Eprod\ ty_Ehreal_E_Ehreal)) \quad (14)$$

Let $c_Erealax_E_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_E_Ereal_ABS_CLASS \in (ty_Erealax_E_Ereal)^{(2^{(ty_Epair_E_Eprod\ ty_Ehreal_E_Ehreal\ ty_Ehreal_E_Ehreal)})} \quad (15)$$

Definition 19 We define $c_Erealax_E_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_E_Eprod\ ty_Ehreal_E_Ehreal\ ty_Ehreal_E_Ehreal)$

Definition 20 We define $c_Erealax_E_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_E_Ereal. (ap\ c_Erealax_E_Ereal.$

Definition 21 We define $c_Ereal_E_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_E_Ereal. \lambda V1y \in ty_Erealax_E_Ereal.$

Definition 22 We define $c_Ebool_E_ECOND$ to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 23 We define $c_Ereal_E_Eabs$ to be $\lambda V0x \in ty_Erealax_E_Ereal. (ap\ (ap\ (ap\ (c_Ebool_E_ECOND$

Definition 24 We define $c_Ereal_topology_E_Ebounded_def$ to be $\lambda V0s \in (2^{ty_Erealax_E_Ereal}). (ap\ (c_Ebo$

Let $c_Eenum_E_EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_E_EREP_num \in (\omega^{ty_Eenum_E_Eenum}) \quad (16)$$

Let $c_Eenum_E_ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_E_ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 25 We define $c_Eenum_E_ESUC$ to be $\lambda V0m \in ty_Eenum_E_Eenum. (ap\ c_Eenum_E_EABS_num$

Definition 26 We define $c_Eprim_rec_E_E3C$ to be $\lambda V0m \in ty_Eenum_E_Eenum. \lambda V1n \in ty_Eenum_E_Eenum.$

Definition 27 We define $c_Earithmic_E_E3E$ to be $\lambda V0m \in ty_Eenum_E_Eenum. \lambda V1n \in ty_Eenum_E_Eenum.$

Definition 28 We define $c_Ebool_E_E5C_E_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_E_E21\ 2) (\lambda V2t \in$

Definition 29 We define $c_Earithmic_E_E3E_E_3D$ to be $\lambda V0m \in ty_Eenum_E_Eenum. \lambda V1n \in ty_Eenum_E_Eenum.$

Let $ty_2Ereal_topology_2Enet : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ereal_topology_2Enet A0) \quad (18)$$

Let $c_2Ereal_topology_2Emk_net : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow c_2Ereal_topology_2Emk_net \\ A_27a \in ((ty_2Ereal_topology_2Enet A_27a)^{(2^{A_27a})^{A_27a}}) \end{aligned} \quad (19)$$

Definition 30 We define $c_2Ereal_topology_2Esequentially$ to be $(ap (c_2Ereal_topology_2Emk_net ty_2Ereal_topology_2Emk_net))$

Definition 31 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27c^{A_27a}).$

Let $c_2Ereal_topology_2Enetord : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ereal_topology_2Enetord A_27a \in ((2^{A_27a})^{A_27a})^{(ty_2Ereal_topology_2Enet A_27a)} \quad (20)$$

Definition 32 We define $c_2Ereal_topology_2Etrivial_limit$ to be $\lambda A_27a : \iota. \lambda V0net \in (ty_2Ereal_topology_2Enetord A_27a).$

Definition 33 We define $c_2Ereal_topology_2Eeventually$ to be $\lambda A_27a : \iota. \lambda V0p \in (2^{A_27a}). \lambda V1net \in (ty_2Ereal_topology_2Etrivial_limit A_27a).$

Definition 34 We define $c_2Ereal_topology_2E_2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda V0f \in (ty_2Erealax_2Ereal^{A_27a}).$

Definition 35 We define $c_2Ereal_topology_2Ecompact$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap (c_2Ebool_2Eand V0s))$

Definition 36 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1s \in (c_2Ereal_topology_2Eeventually V1s).$

Definition 37 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2Eor V1t))$

Definition 38 We define $c_2Ereal_topology_2Eclosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}). (ap (ap (c_2Epred_set_2EUNION V0s)))$

Definition 39 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (c_2Ereal_topology_2Eeventually V1s).$

Definition 40 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Ebool_2Eand V1t))$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (21)$$

Definition 41 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal \\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (22)$$

Definition 42 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 43 We define $c_Ereal_topology_2Elinear$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).(((p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V0s) V1t)) \wedge \\ & (p (ap (ap (c_2Epred_set_2ESUBSET A_27a) V1t) V0s))) \Rightarrow (V0s = V1t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27a}).((p (ap (ap (c_2Epred_set_2ESUBSET \\ & A_27a) V0s) V1t)) \Rightarrow (\forall V2f \in (A_27b^{A_27a}).(p (ap (ap (c_2Epred_set_2ESUBSET \\ & A_27b) (ap (ap (c_2Epred_set_2EIMAGE A_27a A_27b) V2f) V0s)) (\\ & ap (ap (c_2Epred_set_2EIMAGE A_27a A_27b) V2f) V1t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(p (ap (ap (c_2Epred_set_2ESUBSET \\ & ty_2Erealax_2Ereal) V0s) (ap c_2Ereal_topology_2Eclosure V0s)))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).(\forall V1t \in (2^{ty_2Erealax_2Ereal}). \\ & (((p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) V0s) \\ & V1t)) \wedge (p (ap c_2Ereal_topology_2EClosed V1t))) \Rightarrow (p (ap (ap (c_2Epred_set_2ESUBSET \\ & ty_2Erealax_2Ereal) (ap c_2Ereal_topology_2Eclosure V0s) \\ & V1t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Ecompact \\ & V0s)) \Rightarrow (p (ap c_2Ereal_topology_2EClosed V0s)))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned}
& (\forall V0s \in (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Ecompact \\
& (ap c_2Ereal_topology_2Eclosure V0s))) \Leftrightarrow (p (ap c_2Ereal_topology_2Ebounded_def \\
& V0s))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).(((p (ap c_2Ereal_topology_2Ecompact \\
& V1s)) \wedge (p (ap c_2Ereal_topology_2Elinear V0f)))) \Rightarrow (p (ap c_2Ereal_topology_2Ecompact \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0f) V1s))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).((p (ap c_2Ereal_topology_2Elinear \\
& V0f)) \Rightarrow (p (ap (ap (c_2Epred_set_2ESUBSET ty_2Erealax_2Ereal) \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0f) (ap c_2Ereal_topology_2Eclosure V1s))) (ap c_2Ereal_topology_2Eclosure \\
& (ap (ap (c_2Epred_set_2EIMAGE ty_2Erealax_2Ereal ty_2Erealax_2Ereal) \\
& V0f) V1s))))))
\end{aligned} \tag{33}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1s \in \\
& (2^{ty_2Erealax_2Ereal}).(((p (ap c_2Ereal_topology_2Elinear \\
& V0f)) \wedge (p (ap c_2Ereal_topology_2Ebounded_def V1s))) \Rightarrow ((ap \\
& c_2Ereal_topology_2Eclosure (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) V1s)) = (ap (ap (c_2Epred_set_2EIMAGE \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V0f) (ap c_2Ereal_topology_2Eclosure \\
& V1s))))))
\end{aligned}$$