

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(4)

Definition 11 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27b)\ V0s\ V1t)$

Let $c_2Ereal_topology_2EDist : \iota$ be given. Assume the following.

$$c_2Ereal_topology_2EDist \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)})$$
(5)

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal$$
(6)

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})$$
(7)

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ a)\ V0a)$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})$$
(8)

Definition 14 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(ap\ (c_2Etreall_lt\ T1)\ T2)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega$$
(9)

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega})$$
(11)

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$
(12)

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0P)))$

Definition 17 We define $c_2Ereal_topology_2EOpen$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (c_2Ebool_2E2$

Definition 18 We define $c_2Ereal_topology_2EClosed$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap c_2Ereal_topo$

Definition 19 We define $c_2Ereal_topology_2Elimit_point_of$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1s \in ($

Definition 20 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in$

Definition 21 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 22 We define $c_2Ereal_topology_2EClosure$ to be $\lambda V0s \in (2^{ty_2Erealax_2Ereal}).(ap (ap (c_2Epred$

Definition 23 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap ($

Definition 24 We define $c_2Epred_set_2EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_s$

Definition 25 We define $c_2Etopology_2EHull$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).\lambda V1s \in (2^{A_27a}).(ap (c$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((ap c_2Ereal_topology_2EClosure V0s) = (ap (ap (c_2Etopology_2EHull ty_2Erealax_2Ereal) c_2Ereal_topology_2EClosed V0s)))) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}).(\forall V1s \in (2^{A_27a}).((ap (ap (c_2Etopology_2EHull A_27a) V0P) (ap (ap (c_2Etopology_2EHull A_27a) V0P) V1s))) = (ap (ap (c_2Etopology_2EHull A_27a) V0P) V1s)))) \tag{17}$$

Theorem 1

$$(\forall V0s \in (2^{ty_2Erealax_2Ereal}).((ap c_2Ereal_topology_2EClosure (ap c_2Ereal_topology_2EClosure V0s) = (ap c_2Ereal_topology_2EClosure V0s))))$$